Nonlinear Analysis 95 (2014) 743–755

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Existence of traveling waves in van der Waals fluids with viscosity and capillarity effects

Mai Duc Thanh[a,*], Nguyen Dinh Huy[b], Nguyen Huu Hiep[b], Dao Huy Cuong[c]

[a] Department of Mathematics, International University, Quarter 6, Linh Trung Ward, Thu Duc District, Ho Chi Minh City, Viet Nam
[b] Faculty of Applied Science, University of Technology, 268 Ly Thuong Kiet, str. District 10, Ho Chi Minh City, Viet Nam
[c] Nguyen Huu Cau High School, 07 Nguyen Anh Thu, Trung Chanh, Hoc Mon, Ho Chi Minh City, Viet Nam

ARTICLE INFO

Article history:
Received 9 August 2012
Accepted 16 October 2013
Communicated by Enzo Mitidieri

Keywords:
van der Waals fluid
Traveling wave
Shock wave
Viscosity
Capillarity
Lyapunov stability

ABSTRACT

We establish the existence of traveling waves of non-isentropic van der Waals fluids with viscosity and capillarity effects. The method developed the one for simpler models. The nonconvex equation of state of the fluid causes much difficulty in evaluating the related quantities, and so the argument and the analysis are much more involved than the convex equation of state. The point is to estimate the pressure along the Hugoniot curves such that a Lyapunov function can be defined in an appropriate way.

1. Introduction

In this paper we study the existence of traveling waves in van der Waals fluids with the effects of viscosity and capillarity. The viscous–capillary model is given by

$$\begin{aligned} &v_t - u_x = 0,\\ &u_t + p_x = \left(\frac{\lambda}{v}u_x\right)_x - (\mu v_x)_{xx},\\ &E_t + (up)_x = \left(\frac{\lambda}{v}uu_x\right)_x - (u(\mu v_x)_x)_x + (\mu u_x v_x)_x\,, \end{aligned} \tag{1.1}$$

for $x \in \mathbb{R}$ and $t > 0$. As usual, the symbols $\rho, v = 1/\rho, S, p, \varepsilon, T$ and u denote the density, specific volume, entropy, pressure, internal energy, temperature, and velocity, respectively, and

$$E = \varepsilon + \frac{u^2}{2} + \frac{\mu}{2}v_x^2 \tag{1.2}$$

is the total energy. The quantities λ and μ represent the viscosity and capillarity coefficients, respectively. For simplicity, throughout we assume that λ and μ are positive constants. Besides, a van der Waals fluid is characterized by the equation

* Corresponding author. Tel.: +84 8 2211 6965; fax: +84 8 3724 4271.
E-mail addresses: mdthanh@hcmiu.edu.vn, mdthanh1@gmail.com (M.D. Thanh), dinhhuy56@hcmut.edu.vn (N.D. Huy), nguyenhuuhiep47@yahoo.com (N.H. Hiep), cuongnhc82@gmail.com (D.H. Cuong).

0362-546X/$ – see front matter
http://dx.doi.org/10.1016/j.na.2013.10.017

of state of the form

$$p = \frac{RT}{v-b} - \frac{a}{v^2}, \tag{1.3}$$

where $a > 0, b > 0$ and $R > 0$ are constants. Nonclassical (non-Lax) shocks have been known to appear in van der Waals fluids, and they exist at the expenses of Lax shocks by the nucleation criterion, which prefers non-Lax shocks over Lax shocks. Moreover, Riemann solvers with kinetics use both kinds of Lax shocks and of nonclassical (non-Lax) shocks, see [1–3], for example. Traveling waves associated with nonclassical shocks in the Euler equations for van der Waals fluids with viscosity and capillarity effects were obtained by Bedjaoui–LeFloch [4]. The question is whether or not the Euler equations for van der Waals fluids with viscosity and capillarity effects can also possess traveling waves associated with Lax shocks? Our aim in this paper is to seek a positive answer to this question. Therefore, our results indicate that the viscous–capillary models are appropriate for applications involving both kinds of Lax shocks and nonclassical shock waves.

The existence of traveling waves has attracted many authors. Recently, Thanh [5] studied the existence of traveling waves in the Euler equations for polytropic ideal fluids with viscosity and capillarity. So, the result is applied for convex equations of state. In this work, the viscosity and capillarity coefficients are slightly different from the ones in [5]. More interestingly, the fluid is of van der Waals type (1.3). This is a typical nonconvex equation of state. Consequently, the system of Euler equations may be of mixed type as an elliptic–hyperbolic model, and the characteristic fields are not entirely genuinely nonlinear. The analysis and the argument will be much more involved for van der Waals fluids, since they possess complicated features not only in the characteristic fields, but also in the Rankine–Hugoniot relations, and the admissibility criteria for shock waves, etc. However, we can establish the existence of traveling waves of (1.1) for van der Waals fluids when the viscosity and capillarity are suitably chosen. The point is that we can estimate the pressure along the Hugoniot curves such that we can suitably define a Lyapunov function. We will show that the viscous–capillary model (1.1) for van der Waals fluids can still yield nice properties of the corresponding Lyapunov function. Accordingly, the level sets near the saddle point of the Lyapunov function will be proved to provide sharp estimates for the attraction domain of the asymptotically stable equilibrium point. A stable trajectory from the saddle point will then be shown to enter the attraction domain of the asymptotically stable equilibrium point. This saddle-to-stable connection gives us the traveling wave of (1.1).

We observe that traveling waves corresponding to a given non-Lax shock for viscous–capillary models were considered by LeFloch and his collaborators and students, see [6–9,4,10,11]. Traveling waves corresponding to a given Lax shock for viscous–capillary models were obtained by Thanh [12–15,5]. Traveling waves were considered earlier for diffusive–dispersive scalar equations by Bona and Schonbek [16], Jacobs, McKinney, and Shearer [17]. Traveling waves and admissibility criteria of the hyperbolic–elliptic model of phase transition dynamics were also studied by Slemrod [18,19] and Fan [20,21], Shearer and Yang [22]. See also [23–25] for related works.

This paper is organized as follows. In Section 2 we provide basic concepts and properties of the Euler equations for van der Waals fluids. In Section 3 we study the system of ordinary differential equations which are derived from (1.1) for the traveling wave. Its equilibria and the stability of equilibria using linearization will be presented. In Section 4, we define a Lyapunov function, estimate the attraction domain, and establish the existence of the traveling wave.

2. Preliminaries

2.1. Hyperbolicity

Several equations of state of a van der Waals fluid other than (1.3) are given by

$$\begin{aligned}
T &= \frac{d}{(v-b)^{2/3}} \exp\left(\frac{2S}{3R} - \frac{5}{3}\right),\\
\varepsilon &= \frac{3R}{2}T - \frac{a}{v} = \left(p + \frac{a}{v^2}\right)\frac{3(v-b)}{2} - \frac{a}{v},
\end{aligned}$$

where $d > 0$ is a parameter, see [26]. Substituting T from the last system into (1.3), one obtains an equation of state of the form $p = p(v, S)$, where

$$p(v,S) = \frac{Rd}{(v-b)^{5/3}} \exp\left(\frac{2S}{3R} - \frac{5}{3}\right) - \frac{a}{v^2}.$$

See Fig. 1.

Consider the fluid dynamics equations in the Lagrangian coordinates

$$\begin{aligned}
&v_t - u_x = 0,\\
&u_t + p_x = 0,\\
&E_t + (up)_x = 0, \quad x \in \mathbb{R}, t > 0.
\end{aligned} \tag{2.1}$$