Nonlinear Analysis 90 (2013) 86–95

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Pullback attractors of a multi-valued process generated by parabolic differential equations with unbounded delays

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ARTICLE INFO

Article history:
Received 21 August 2012
Accepted 24 May 2013
Communicated by S. Carl

MSC:
35R10
35B41

Keywords:
Multi-valued process
Pullback attractor
Parabolic differential equation
Infinite delay
Flattening

ABSTRACT

The theory of nonautonomous multi-valued dynamical systems and a method of asymptotic compactness based on the concept of the Kuratowski measure of the noncompactness of a bounded set are used to prove the existence of pullback attractors in the weighted space $C_{\gamma,V}$ for the multi-valued process associated with the nonautonomous parabolic differential equation with infinite delay for which the uniqueness of solutions need not hold.

1. Introduction

The asymptotic behavior of solutions to the following nonautonomous parabolic differential equation with infinite delays defined on a bounded domain Ω in $\mathbb{R}^n$ with smooth boundary is investigated. The equations have the form

$$\frac{\partial u}{\partial t} + Au + bu = f(t, u_t) + g(t, x),$$

where A is a densely-defined self-adjoint positive linear operator with domain $D(A) \subset L^2(\Omega)$ and with compact resolvent, the constant $b \geq 0$, the nonlinear term

$$f(t, u_t(t, x)) = F(t, u(t - \rho(t), x)) + \int_{-\infty}^{0} G(t, s, u(t + s, x))ds,$$

and g is a nonautonomous external force satisfying certain conditions. It is not assumed that the associated initial–boundary value problems (see (1) below) have unique solutions. Retarded differential equations of this type arise in many physical systems of non-instantaneous transmission phenomena such as high velocity fields in wind tunnel experiments as well as in special biological situations involving population growth or the incubating time on disease models, e.g., [1,2].

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http://dx.doi.org/10.1016/j.na.2013.05.026

The qualitative theory of global attractors for autonomous delay systems is now well-established, see, e.g., [3–6]. The existence of attractors for the models containing hereditary characteristics with bounded and unbounded delays has been investigated for instance in [7–10] and the references therein. In recent years, the asymptotic behavior of multi-valued nonautonomous infinite-dimensional dynamical systems has attracted much attention in mathematical literature, see, e.g., [7,10–16] and the references therein. Here we prove the existence of a nonautonomous pullback attractor in a space of higher regularity for the above nonautonomous parabolic differential equation with infinite delays.

The main aim of this paper is three-fold. Firstly, we give sufficient conditions for the existence of pullback attractors for a multi-valued process and investigate the relation between pullback attractors with different attraction properties is investigated. Moreover, we present a new method to check the asymptotical upper-semicompactness of the multi-valued dynamical systems with infinite delays. Secondly, we establish the existence of bounded absorbing set. Here we borrow some ideas from [7,17] to deal with the unbounded delay term and overcome some difficulties caused by the infinite dimensional case, respectively. Finally, we consider the existence of a pullback attractor in a weighted space $C_{\gamma,V}$ for the nonautonomous parabolic differential equation with infinite delays and without the uniqueness of solutions. In particular, we verify the asymptotical upper-semicompactness of the solutions by decomposing the infinite delay into two parts, with the finite part handled directly with methods used in [17]. In addition, we prove the invariance of pullback attractors for the multi-valued process associated with the unbounded delay equation.

The paper is organized as follows. Section 2 gives some preliminary results and definitions, while in Sections 3 and 4 the problem is set in a suitable nonautonomous framework and the existence of bounded absorbing sets is established. Finally, in Section 5 the existence of a pullback attractor in the weighted space $C_{\gamma,V}$ is proved.

2. Multi-valued processes

Let X be a complete metric space with metric $d_X(\cdot,\cdot)$, and let $\mathscr{P}(X)$ be the class of nonempty subsets of X. Denote by $H_X^*(\cdot,\cdot)$ the Hausdorff semidistance between two nonempty subsets of a complete metric space X, which are defined by

$$H_X^*(A,B) = \sup_{a\in A} \operatorname{dist}_X(a,B),$$

where $\operatorname{dist}_X(a,B) = \inf_{b\in B} d_X(a,b)$.

Definition 1. A family of mappings $U(t,\tau) : X \to \mathscr{P}(X)$, $t \geq \tau$, $\tau \in \mathbb{R}$, is called to be a multi-valued process (MVP in short) if

(1) $U(\tau,\tau)x = \{x\},\ \forall \tau \in \mathbb{R},\ x \in X$;
(2) $U(t,s)U(s,\tau)x = U(t,\tau)x,\ \forall t \geq s \geq \tau,\ \tau \in \mathbb{R},\ x \in X$.

The Kuratowski measure $k(A)$ of noncompactness of set A is defined by

$$k(A) = \inf\{\delta > 0 \mid A \text{ admits a finite cover by sets whose diameter } \leq \delta\}.$$

Definition 2. Let $\{U(t,\tau)\}$ be a multi-valued process on X. We say that $\{U(t,\tau)\}$ is

(1) pullback dissipative, if there exists a family of bounded sets $\mathscr{D} = \{D(t)\}_{t\in\mathbb{R}}$ in X so that for any bounded set $B \subset X$ and each $t \in \mathbb{R}$, there exists a $t_0 = t_0(B,t) \in \mathbb{R}^+$ such that

$$U(t,t-s)B \subset D(t), \quad \forall s \geq t_0;$$

(2) $\mathscr{D}$-pullback ω-limit compact with respect to each $t \in \mathbb{R}$, if for any $\varepsilon > 0$, there exists a $t_1 = t_1(\mathscr{D},t,\varepsilon) \in \mathbb{R}^+$ such that

$$k\left(\bigcup_{s\geq t_1} U(t,t-s)D(t-s)\right) \leq \varepsilon.$$

A multi-valued process $\{U(t,\tau)\}$ is said to be $\mathscr{D}$-pullback asymptotically upper-semicompact in X if for each fixed $t \in \mathbb{R}$, any sequence $\{T_n\}$ with $T_n \to +\infty$, $\{x_n\}$ with $x_n \in D(t-T_n)$, and $\{y_n\}$ with $y_n \in U(t,t-T_n)x_n$, this last sequence $\{y_n\}$ is relatively compact in X.

Theorem 3 ([16]). *Let $\{U(t,\tau)\}$ be a multi-valued process on X. Then $\{U(t,\tau)\}$ is $\mathscr{D}$-pullback asymptotically upper-semicompact if and only if $\{U(t,\tau)\}$ is $\mathscr{D}$-pullback ω-limit compact.*

Theorem 4. *If the multi-valued process $\{U(t,\tau)\}$ is $\mathscr{D}$-pullback ω-limit compact in X, then $\{U(t,\tau)\}$ is pullback ω-limit compact for any bounded subset B of X.*