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Local well-posedness and stability of solitary waves for the two-component Dullin–Gottwald–Holm system

Xingxing Liu [a,b,*], Zhaoyang Yin [b,c]

[a] Department of Mathematics, China University of Mining and Technology, 221116 Xuzhou, China
[b] Department of Mathematics, Sun Yat-sen University, 510275 Guangzhou, China
[c] Faculty of Information Technology, Macau University of Science and Technology, Macau, China

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ABSTRACT

In this paper, we study the Cauchy problem for the two-component Dullin–Gottwald–Holm system with the initial data $(u_0, \rho_0) \in B^s_{p,r}(\mathbb{R}) \times B^{s-1}_{p,r}(\mathbb{R})$, $1 \le p, r \le +\infty$, and $s > \max\{1+\frac{1}{p}, \frac{3}{2}, 2-\frac{1}{p}\}$, which generalizes some previous local well-posed results in Sobolev spaces. Then we prove that all the smooth solitary wave solutions are orbitally stable under small disturbances.

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1. Introduction

Dullin et al. [1] presented the following Dullin–Gottwald–Holm (DGH) equation:

$$u_t + c_0 u_x + 3uu_x - \alpha^2(u_{txx} + uu_{xxx} + 2u_x u_{xx}) + \gamma u_{xxx} = 0, \tag{1.1}$$

with $x \in \mathbb{R}$, $t \ge 0$, where α^2 and $\frac{\gamma}{c_0}$ are squares of length scales, and $c_0 = \sqrt{gh}$ (where $c_0 = 2\omega$) is the dispersion relation for irrotational water waves propagating at the free surface of a layer of water of average depth h, over a flat bed [2,3]. Eq. (1.1) was derived by asymptotic expansions directly in the Hamiltonian for Euler's equation in the shallow water regime [1]. It has a bi-Hamiltonian structure and a Lax pair formulation [4,1]. It is also completely integrable and its traveling wave solutions contain both the solitons for the KdV equation and the peakons for the Camassa–Holm (CH) equation [4] as limiting cases. For the peakon solution, we know that it replicates a feature that is characteristic for the waves of great height-waves of largest amplitude that are exact solutions of the governing equations for water waves [5–8].

Indeed, Eq. (1.1) has been studied in a lot of papers. Here we give a brief review. Tian et al. [9] studied the local well-posedness and the scattering problem of the DGH equation. Yin [10] obtained the global strong solutions and solutions which blow up in finite time. In [11], Zhang et al. proved the existence of global weak solutions to Eq. (1.1) provided that the initial data satisfies some certain conditions. Using the bifurcation method, Guo et al. [12] obtained some expression of

* Corresponding author at: Department of Mathematics, China University of Mining and Technology, 221116 Xuzhou, China. Tel.: +86 15913114645.
E-mail addresses: liuxingxing123456@163.com (X. Liu), mcsyzy@mail.sysu.edu.cn (Z. Yin).

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peakons of Eq. (1.1). When the dispersive parameter $\gamma = -2\alpha^2\omega$, Eq. (1.1) has peaked solitary waves given by the formula $\varphi(x,t) = (c-2\omega)e^{\frac{-|x-ct|}{|\alpha|}}$. In this situation, the orbital stability of one single solitary wave has been proved by Hakkaev [13]. As an extending result, Liu et al. [14] obtained the local well-posedness by Kato's semigroup theory and proved the stability of peakons for a generalized DGH equation. Inspired by [15], Liu et al. [16] established the orbital stability of the train of N solitary waves for the DGH equation. Moreover, Zhang [17] gave some general expression of peakons and Ouyang et al. [18] proved the orbital stability of the general peakons for Eq. (1.1) by the observation of relationship between the CH equation and the DGH equation.

Motivated by the interest in the study of Eq. (1.1), Zhu et al. [19] derived a generalization of the DGH equation from Euler's equation with constant vorticity in shallow water waves moving over a linear shear flow, by using the method shown in [20]. For the important role of the vorticity in the study of water wave theory, we refer the readers to [20,21] for more details. We here consider the following two-component Dullin–Gottwald–Holm (2-DGH) system:

$$\begin{cases} u_t - u_{txx} - Au_x + 3uu_x + \gamma u_{xxx} = 2u_x u_{xx} + uu_{xxx} - \rho\rho_x, & t>0, x\in\mathbb{R}, \\ \rho_t + (\rho u)_x = 0, & t>0, x\in\mathbb{R}, \\ u(0,x) = u_0(x), & x\in\mathbb{R}, \\ \rho(0,x) = \rho_0(x), & x\in\mathbb{R}, \end{cases} \tag{1.2}$$

where $u(t,x)$ represents the horizontal velocity of the fluid, $\rho(t,x)$ is related to the free surface elevation from equilibrium (or scalar density), and the parameter A, $\gamma\in\mathbb{R}$ are constant. From the derivation of System (1.2), the boundary assumptions $u\to 0$, $\rho\to 1$ as $|x|\to\infty$ are required [19,22,23]. Similar to the DGH equation, System (1.2) is also completely integrable and it can be written as a compatibility condition of two linear systems (Lax pair) with a spectral parameter ζ [19,22]:

$$\Psi_{xx} = \left[-\zeta^2\rho^2 + \zeta\left(m - \frac{A}{2} + \frac{\gamma}{2}\right) + \frac{1}{4}\right]\Psi, \qquad \Psi_t = \left(\frac{1}{2\zeta} - u + \gamma\right)\Psi_x + \frac{1}{2}u_x\Psi,$$

where $m = u - u_{xx}$.

Recently, Zhu et al. [19] have studied the well-posedness of the periodic 2-DGH system on the unit circle $\mathbb{S}\triangleq\mathbb{R}/\mathbb{Z}$ by Kato's semigroup theory with the initial data $(u_0,\rho_0)\in H^s(\mathbb{S})\times H^{s-1}(\mathbb{S})$, $s\geq 2$, and have derived the precise blow-up mechanism and the wave-breaking result for System (1.2). In [22], Guo et al. deal with System (1.2) on the line $\mathbb{R}$ in the aspects of the well-posedness, wave-breaking criteria and global strong solutions. It is noted that using the bi-linear estimate technique to the approximate solutions, Chen et al. [24] establish the local well-posedness result for System (1.2) with the initial data $(u_0,\rho_0)\in H^s(\mathbb{R})\times H^{s-1}(\mathbb{R})$, $s>\frac{3}{2}$, which improves the well-posed result shown in [22].

Inspired by [25,26], the goal of the present paper is to address the Cauchy problem of System (1.2) with the initial data $(u_0,\rho_0)\in B^s_{p,r}(\mathbb{R})\times B^{s-1}_{p,r}(\mathbb{R})$, $1\leq p,r\leq+\infty$, and $s>\max\left\{1+\frac{1}{p},\frac{3}{2},2-\frac{1}{p}\right\}$. Since the nonhomogeneous Besov spaces contain Sobolev spaces, i.e., $B^s_{2,2}(\mathbb{R}) = H^s(\mathbb{R})$, thus the results of [22,24] come up as a special case of our results. In particular, we handle the critical index cases $s = 2+\frac{1}{p}, 3+\frac{1}{p}$, when using the useful transport equation theory, by the interpolation method. Moreover, we will explain that the index $s=\frac{3}{2}$ is critical in dealing with the well-posedness of System (1.2) with initial data in Besov spaces $B^{\frac{3}{2}}_{2,r}\times B^{\frac{1}{2}}_{2,r}$ in some sense (see Remark 3.2). Another purpose of the paper is to prove that all smooth solitary wave solutions are orbitally stable in the energy spaces $H^1(\mathbb{R})\times L^2(\mathbb{R})$. Although we know that an excellent proof of stability of peakons for the CH equation is given by Constantin and Strauss [27] using the conservation laws, however, for this coupled two-component system (1.2), we find it is difficult to use that method here. Fortunately, we find that the problem can be solved by the general approach to the orbital stability of a Hamiltonian system introduced by Grillakis et al. in [28].

The remainder of paper is organized as follows. In Section 2, we present some facts on Besov spaces, some preliminary properties and the transport equation theory. In Section 3, we establish the local well-posedness result of System (1.2) in Besov spaces. In Section 4, we are devoted to the orbital stability of the solitary wave solutions of System (1.2).

Notation. In the following, we denote $C>0$ as a generic constant only depending on p,r,s. Since our discussion about System (1.2) is on the line $\mathbb{R}$, for simplicity, we omit $\mathbb{R}$ in our notations of function spaces. And we denote the Fourier transform of a function u as $\mathcal{F}u$. All the transpose of a vector $\vec{f}$ or a sequence of vectors $\{\vec{f}_n\}_{n\in\mathbb{N}}$ is presented as $\vec{f}^t$ or $\{(\vec{f}_n)^t\}_{n\in\mathbb{N}}$.

2. Preliminaries

In this section, we will recall some basic theory of the Littlewood–Paley decomposition and the transport equation theory on Besov spaces, which will play an important role in the sequel. One may get more details from [29,30].

Proposition 2.1 (*Littlewood–Paley Decomposition [29]*). *Let $\mathcal{B}\triangleq\left\{\xi\in\mathbb{R}, |\xi|\leq\frac{4}{3}\right\}$ and $\mathcal{C}\triangleq\left\{\xi\in\mathbb{R}, \frac{3}{4}\leq|\xi|\leq\frac{8}{3}\right\}$. Then there exist $\chi(\xi)\in C_c^\infty(\mathcal{B})$ and $\eta(\xi)\in C_c^\infty(\mathcal{C})$ such that*

$$\chi(\xi) + \sum_{q\in\mathbb{N}}\eta(2^{-q}\xi) = 1, \quad \forall\,\xi\in\mathbb{R}$$

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