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Asymptotics of solutions for a basic case of fluid–structure interaction

Christoph Boeckle *, Peter Wittwer

Theoretical Physics Department, University of Geneva, Switzerland

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ABSTRACT

We consider the Navier–Stokes equations in a half-plane with a drift term parallel to the boundary and a small source term of compact support. We provide detailed information on the behavior of the velocity and the vorticity at infinity in terms of an asymptotic expansion at large distances from the boundary. The expansion is universal in the sense that it only depends on the source term through some constants. The expansion also applies to the problem of an exterior flow past a small body moving at constant velocity parallel to the boundary, and can be used as an artificial boundary condition on the edges of truncated domains for numerical simulations.

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1. Introduction

In this paper, we study the steady Navier–Stokes equations in the half-plane $\Omega_+ = \left\{(x, y) \in \mathbb{R}^2 \mid y > 1\right\}$ with a drift term parallel to the boundary, a force of compact support, and zero Dirichlet boundary conditions at the boundary of the half-plane and at infinity,

$$\partial_x \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla p - \Delta \boldsymbol{u} = \boldsymbol{F}, \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{F}$ is smooth and of compact support in Ω_+, *i.e.*, $\boldsymbol{F} \in C_c^\infty(\Omega_+)$, subject to the boundary conditions

$$\boldsymbol{u}(x, 1) = 0, \quad x \in \mathbb{R}, \tag{3}$$

$$\lim_{\mathbf{x}\to\infty} \boldsymbol{u}(\mathbf{x}) = 0. \tag{4}$$

Our main result is an asymptotic expansion for the solution to this problem.

For small forces, existence of a solution for this system together with basic bounds on the decay at infinity was proved in [1], and uniqueness of solutions was proved in [2] in a very general context. In [3] additional information on the decay at infinity was obtained. In a similar three dimensional case (see [4]), the asymptote of the velocity field has been analyzed to leading order. For a general introduction to the method used in this series of papers, see [5].

In [2] it was also shown that the asymptotic behavior of the unique solution to (1)–(4) is identical to the one for the problem of an exterior flow without force past a small body $\mathcal{B}$ moving parallel to the wall at constant velocity described in a frame comoving with the body. See Fig. 1 for a schematic of the two related problems. As a consequence, the asymptotic expansion which we provide here also describes the asymptotic behavior of the solution for the problem with a body. This asymptotic expression turns out to be particularly useful in order to provide artificial boundary conditions for numerical simulations of the flow with a body; see [6]. For completeness, we note that artificial boundary conditions obtained this way have also been applied with success in the numerical resolution of two and three-dimensional flows in the full space (see [7–10]).

* Corresponding author.
E-mail addresses: christoph.boeckle@unige.ch (C. Boeckle), peter.wittwer@unige.ch (P. Wittwer).

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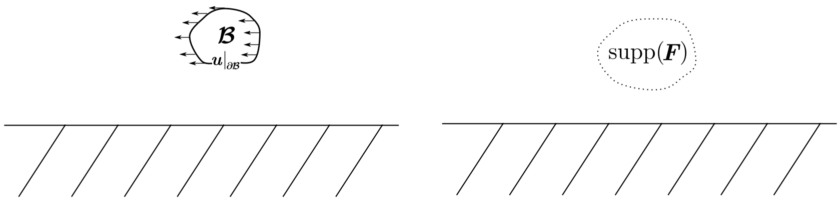

Fig. 1. Left: the problem with a small body moving at constant velocity parallel to a wall. Right: the related problem without a body, but with a force term of compact support, studied in this paper.

Whereas the efforts in previous papers were concentrated on proving various basic properties of the solution, the present paper focuses on obtaining the above-mentioned explicit asymptotic expansion. In the remainder of this paper, when we invoke "the solution", it will be understood that it has the properties proved in the previous papers referenced above (*i.e.*, existence, uniqueness and decay properties).

Our main result is summarized in the following theorem.

Theorem 1. *Let $\boldsymbol{u} = (u, v)$ and p be the solution to Eqs. (1)–(4) for $\boldsymbol{F}$ small and let ω be the vorticity. Then, there exist constants c_1, c_2 such that for $\varepsilon > 0$,*

$$\lim_{y\to\infty} \sup_{x\in\mathbb{R}} |y^{5/2-\varepsilon}(u(x,y) - u_{as}(x,y))| = 0, \tag{5}$$

$$\lim_{y\to\infty} \sup_{x\in\mathbb{R}} |y^{5/2-\varepsilon}(v(x,y) - v_{as}(x,y))| = 0, \tag{6}$$

$$\lim_{y\to\infty} \sup_{x\in\mathbb{R}} |y^{9/2-\varepsilon}(\omega(x,y) - \omega_{as}(x,y))| = 0, \tag{7}$$

with

$$u_{as}(x,y) = \frac{c_1}{y^{3/2}}\varphi_1(x/y) + \frac{c_1}{y^2}\varphi_{2,1}(x/y) + \frac{c_2}{y^2}\varphi_{2,2}(x/y) - \frac{c_1}{y^2}\eta_W(x/y^2) - \frac{c_1}{y^3}\eta_B(x/y^2), \tag{8}$$

$$v_{as}(x,y) = \frac{c_1}{y^{3/2}}\psi_1(x/y) + \frac{c_1}{y^2}\psi_2(x/y) + \frac{c_2}{y^2}\psi_{2,2}(x/y) + \frac{c_1}{y^3}\omega_W(x/y^2) + \frac{c_1}{y^4}\omega_B(x/y^2), \tag{9}$$

$$\omega_{as}(x,y) = \frac{c_1}{y^3}\omega_W(x/y^2) + \frac{c_1}{y^4}\omega_B(x/y^2), \tag{10}$$

and functions $\varphi_1, \varphi_{2,1}, \varphi_{2,2}, \psi_1, \psi_{2,1}, \psi_{2,2}, \eta_W, \eta_B, \omega_W$ and ω_B as given in Appendix A.1.

Remark 2. This theorem is an immediate consequence of Theorem 10 in Section 3.

- The functions $\varphi_1, \varphi_{2,1}, \varphi_{2,2}, \psi_1, \psi_{2,1}, \psi_{2,2}, \eta_W, \eta_B, \omega_W$ and ω_B are universal, *i.e.*, independent of $\boldsymbol{F}$.
- The power $5/2$ in the limits (5) and (6) is sharp, whereas the power $9/2$ in (7) can probably be improved by $1/2$ at the price of additional computations.
- Some terms in (8) and (9) are unimportant in view of the limits (5) and (6), but they are included such as to form a divergence-free velocity field in pairs of successive terms of u_{as} and v_{as} and such as to have two orders in both of the two scalings x/y and x/y^2.
- The explicit forms of u_{as} and v_{as} imply that

  $$\lim_{y\to\infty} y^{3/2}u(xy,y) = c_1\varphi_1(x),$$

  $$\lim_{y\to\infty} y^{3/2}v(xy,y) = c_1\psi_1(x),$$

  which shows that the bounds given in [1] are sharp. Remark that this decay is faster than in the full plane (see [7]). Moreover, the components of the velocity field associated to the functions φ_i and ψ_i are harmonic. The asymptotic expansion is thus given by the superposition of a potential flow and a flow carrying the vorticity, which is concentrated, to leading order, in a parabolic region called the "wake", in the sense that

  $$\lim_{y\to\infty} y^3\omega_{as}(xy^2,y) = c_1\omega_W(x).$$

  In contrast to the case of an exterior problem in $\mathbb{R}^2$ (see for example [8] or [11, pp. 826–829]), the vorticity is however not exponentially small outside the wake, since we have in particular, for all $x \in \mathbb{R}$,

  $$\lim_{y\to\infty} y^4\omega_{as}(x,y) = c_1\omega_B(0) \neq 0,$$

  which shows that a background of vorticity is created by the interaction of the fluid with the boundary.

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