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Large deviation for the nonlocal Kuramoto–Sivashinsky SPDE

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ABSTRACT

In this paper, we establish a large deviation principle for the (weak) solution to a nonlocal Kuramoto–Sivashinsky stochastic partial differential equation with small noise perturbation. The key technique is an application of the contraction principle.

1. Introduction

It is known that the deterministic 1-dimensional Kuramoto–Sivashinsky (K–S) equation,

$$du(t) + (u_{xxxx} + u_{xx} + uu_x)\,dt = 0 \tag{1}$$

arises in the modelling of the flow of a thin film of viscous liquid falling down an inclined plane, subject to an applied electric field. With an impact of a nonlocal term, Duan and Vincent [1] studied the dynamics concerning the deterministic nonlocal K–S equation. In a successive paper [2], the authors discussed a stochastic version of Eq. (1) with an additive white noise. They proved that a unique weak solution exists in $L^4(0,T;L^4(G))$, $\mathbb{P}$-a.s. for the equation with homogeneous Dirichlet boundary conditions. In [3], Yang discussed an analogous subject as in [1] for the equation driven by an additive white noise with the impact of the nonlocal term, which is described in the following form:

$$\begin{cases} du(t) + (u_{xxxx}(t) + u_{xx}(t) + u(t)u_x(t))\,dt + \alpha\mathcal{H}\mathcal{I}(u_{xxx}(t))\,dt = \sigma\,dW(t), & \text{in } G,\\ u \text{ is periodic on } G := (-\ell,\ell), \quad \text{i.e., } u(x+\ell) = u(x-\ell) \text{ for } x\in G, \end{cases} \tag{2}$$

where the constants $\ell > 0$, $\alpha > 0$, $\sigma \in \mathbb{R}$ and the nonlocal term $\mathcal{H}\mathcal{I}(u)$ is the Hilbert transform given by

$$\mathcal{H}\mathcal{I}(u)(x) := -\frac{1}{2\ell}\int_{-\ell}^{\ell} \cot\frac{\pi(x-y)}{2\ell}\,u(y)dy, \quad x\in G. \tag{3}$$

The noise term in (2) is an additive noise σdW, where $W = (W(t);\ 0\le t\le 1)$ is a Q-Wiener process on a complete probability space $(\Omega,\mathcal{F},(\mathcal{F}_t;\ t\ge 0),\mathbb{P})$, where the filtration $(\mathcal{F}_t;\ t\ge 0)$ satisfies the usual conditions.

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In this paper, we are concerned with a large deviation principle (LDP) for the (weak) solution $u^\varepsilon = (u^\varepsilon(t);\ 0 \le t \le 1)$ to Eq. (2) with small noise perturbations. Namely, for any $\varepsilon > 0$ and $t \in [0, 1]$, the $(\mathcal{F}_t;\ t \ge 0)$-adapted process u^ε is governed by

$$\begin{cases} du^\varepsilon(t) + \left(u^\varepsilon_{xxxx}(t) + u^\varepsilon_{xx}(t) + u^\varepsilon(t)u^\varepsilon_x(t)\right)dt + \alpha\mathcal{H}\mathcal{I}\left(u^\varepsilon_{xxx}(t)\right)dt = \varepsilon\sigma dW(t), & \text{in } G; \\ u^\varepsilon \text{ is periodic on } G. \end{cases} \tag{4}$$

The LDP for various stochastic partial differential equations (SPDEs) driven by white noise have been studied in the literature (see, e.g., [4–6] and references therein). In [4], Cardon-Weber obtained the LDP for the 1-dimensional stochastic Burgers type equation by proving the uniform Freidlin–Wentzell estimates. Further, Carreras and Sarrà studied the LDP for a d-dimensional stochastic heat equation with spatially correlated noise in [5]. Recently, Röckner et al. [6] established the LDP for stochastic generalized porous media equations using the generalized contraction principle (see Theorem 3.2 therein). Motivated by the idea employed in [6], we prove the LDP of the (weak) solution to Eq. (2) with small noise perturbations (namely, the solution to Eq. (4) for $\varepsilon > 0$) by adopting a version of the contraction principle (see Theorem 4.2.23 in [7]). Note that this technique has been applied to derive the corresponding LDP for diffusion processes or delay SDEs (see, e.g. [7,8]).

The rest of this paper is organized as follows. In Section 2, some preliminaries are given. Section 3 is devoted to establishing probability properties of the (weak) solution to Eq. (4). In Section 4, we explore the skeleton equation corresponding to Eq. (4). Finally a LDP of the (weak) solution to the nonlocal K–S SPDE with small noise perturbation is established in Section 5.

2. Preliminaries

This section would introduce some basic notation, function spaces and functional inequalities used frequently in the paper.

First, we recall a basic fact on the solution to Eq. (2) with the initial value u_0 when the diffusive coefficient $\sigma = 0$, namely, the spatial average $\bar{u}_0$ of u,

$$\bar{u}_0 := \frac{1}{2\ell}\int_{-\ell}^{\ell} u(t,x)dx = \frac{1}{2\ell}\int_{-\ell}^{\ell} u_0(x)dx, \quad \forall\, t \ge 0.$$

W.L.G., suppose that $\bar{u}_0 = 0$ throughout the paper. Thus, we can define the following function spaces,

$$\begin{cases} H := \left\{u \in L^2(G);\ u \text{ is periodic on } G,\ \displaystyle\int_{-\ell}^{\ell} u(x)dx = 0\right\}, \\ H^p_{\text{per}} := \left\{u \in W^{2,p}(G);\ u \text{ is periodic on } G\right\}, \quad \text{for } p \in \mathbb{N}, \\ \dot{H}^p_{\text{per}} := H^p_{\text{per}} \cap H, \quad \text{for } p \in \mathbb{N}. \end{cases}$$

For $i \in \mathbb{N} \cup \{0\}$, let $D_i := \frac{\partial^i}{\partial x^i}$ and $D_0 = I$ (the identity operator on H). Then $A = -D_2$ is a positive self-adjoint unbounded linear operator with domain $D(A)$. Let $(\lambda_k)_{k\in\mathbb{N}}$ and $(e_k)_{k\in\mathbb{N}} := (\phi_k(x), \psi_k(x))_{k\in\mathbb{N}}$ be the eigenvalues and corresponding eigenfunctions of $A : D(A) \to H$. Then, it holds that

$$\begin{cases} \lambda_k = \dfrac{\pi^2 k^2}{\ell^2}, \\ \phi_k(x) = \dfrac{1}{\sqrt{\ell}}\sin\left(\dfrac{k\pi x}{\ell}\right), \\ \psi_k(x) = \dfrac{1}{\sqrt{\ell}}\cos\left(\dfrac{k\pi x}{\ell}\right), \end{cases}$$

and $(e_k)_{k\in\mathbb{N}}$ forms a complete orthonormal basis of H. By the properties of the operator A, for $s \in \mathbb{R}$, the spectral theory allows us to define the powers A^s of A by (see [9])

$$\begin{cases} D(A^s) = \left\{u \in H;\ \displaystyle\sum_{k=1}^{\infty} \lambda_k^{2s}(u, e_k)^2 < \infty\right\}, \\ A^s u = \displaystyle\sum_{k=1}^{\infty} \lambda_k^s (u, e_k)\, e_k, \quad \text{for } u \in D(A^s), \end{cases}$$

where $(\cdot,\cdot)$ and $|\cdot|$ denote the inner product and the corresponding norm of H. We endow the domain $D(A^s)$ of $A^s : D(A^s) \to H$ with the following inner product and the norm

$$\begin{cases} (u, v)_{2s} = \left(A^s u, A^s v\right), \\ |u|_{2s} = \left|A^s u\right|, \end{cases}$$