Nonlinear Analysis 79 (2013) 140–155

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Measure functional differential equations with infinite delay

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ARTICLE INFO

Article history:
Received 15 September 2012
Accepted 17 November 2012
Communicated by Enzo Mitidieri

MSC:
34K05
34G20
34K45
34N05

Keywords:
Measure functional differential equations
Generalized ordinary differential equations
Differential equations in Banach spaces
Regulated functions
Kurzweil–Stieltjes integral
Phase space

ABSTRACT

We introduce measure functional differential equations with infinite delay and an axiomatically described phase space. We show how to transform these equations into generalized ordinary differential equations whose solutions take values in a suitable infinite-dimensional Banach space. Even in the special case of functional equations with finite delay, our result improves the existing one by imposing weaker conditions on the right-hand side.

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1. Introduction

Measure functional differential equations with finite delay have the form

$$y(t) = y(t_0) + \int_{t_0}^{t} f(y_s, s)\,\mathrm{d}g(s), \tag{1.1}$$

where y is an unknown function with values in $\mathbb{R}^n$ and the symbol y_s denotes the function $y_s(\tau) = y(s+\tau)$ defined on $[-r, 0]$, $r \geq 0$ being a fixed number corresponding to the length of the delay. The integral on the right-hand side of (1.1) is the Kurzweil–Stieltjes integral with respect to a nondecreasing function g (see the definition in Section 3; this integral includes the Lebesgue–Stieltjes integral with respect to the measure generated by g).

Measure functional differential equations have been introduced in the paper [1]. In the special case $g(s) = s$, Eq. (1.1) reduces to the classical functional differential equation

$$y(t) = y(t_0) + \int_{t_0}^{t} f(y_s, s)\,\mathrm{d}s, \tag{1.2}$$

which has been studied by many authors (see e.g. [2]). On the other hand, the general form (1.1) includes other familiar types of equations such as functional differential equations with impulses or functional dynamic equations on time scales

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doi:10.1016/j.na.2012.11.018

(see [1,3]). For example, consider the impulsive functional differential equation

$$\left.\begin{aligned} &y'(t) = f(y_t, t), \quad t \in [t_0, \infty) \setminus \{t_1, t_2, \ldots\}, \\ &\Delta^+ y(t_i) = I_i(y(t_i)), \quad i \in \mathbb{N}, \end{aligned}\right\} \tag{1.3}$$

where the impulses take place at preassigned times $t_1, t_2, \ldots \in [t_0, \infty)$, and their action is described by the operators $I_i : \mathbb{R}^n \to \mathbb{R}^n$, $i \in \mathbb{N}$; the solution is assumed to be left-continuous at every point t_i. The corresponding integral form is

$$y(t) = y(t_0) + \int_{t_0}^{t} f(y_s, s)\,\mathrm{d}s + \sum_{i \in \mathbb{N},\, t_i < t} I_i(y(t_i)), \quad t \in [t_0, \infty),$$

which is equivalent (see [3, Lemma 2.4]) to the measure functional differential equation

$$y(t) = y(t_0) + \int_{t_0}^{t} \tilde{f}(y_s, s)\,\mathrm{d}g(s), \quad t \in [t_0, \infty),$$

where $g(s) = s + \sum_{i=1}^{\infty} \chi_{(t_i,\infty)}(s)$ and

$$\tilde{f}(x, s) = \begin{cases} f(x, s), & s \in [t_0, \infty) \setminus \{t_1, t_2, \ldots\}, \\ I_i(x(0)), & s = t_i, \ \text{for } i \in \mathbb{N}. \end{cases}$$

The aim of this paper is to discuss measure functional differential equations with infinite delay, i.e., equations of the form (1.1), where y_s now denotes the function $y_s(\tau) = y(s + \tau)$ defined on $(-\infty, 0]$. The case $g(s) = s$ corresponds to classical functional differential equations with infinite delay, which have been studied by numerous authors (see e.g. [4–6] and the references in these works). The general case when g is a nondecreasing function includes certain other types of functional equations, such as the impulsive functional differential equation (1.3) with infinite delay. One particular example is the impulsive Volterra integro-differential equation

$$y'(t) = \int_0^t a(y(s), s)\,\mathrm{d}s, \quad t \in [0, \infty) \setminus \{t_1, t_2, \ldots\},$$

$$\Delta^+ y(t_i) = I_i(y(t_i)), \quad i \in \mathbb{N},$$

which has the form (1.3) with

$$f(x, t) = \int_{-t}^{0} a(x(\tau), t + \tau)\,\mathrm{d}\tau$$

for a function $x : (-\infty, 0] \to \mathbb{R}^n$.

When dealing with infinite delay, the crucial problem is the choice of the phase space, i.e., the domain of the first argument of f. In the classical case (1.2), the elements of this phase space are continuous functions. Such a phase space is no longer suitable for a general measure functional differential equation, whose solutions are discontinuous functions. The problem of the choice of phase space is discussed in Section 2. We do not restrict ourselves to a particular phase space; instead, we introduce a certain system of conditions and allow the phase space to be any space satisfying these conditions. A similar axiomatic approach was used by various authors (see e.g. [4–6] and the references there) to describe the phase space of classical or impulsive functional differential equations with infinite delay.

In Section 3, we show that under certain natural assumptions, a measure functional differential equation can be transformed to a generalized ordinary differential equation whose solutions take values in an infinite-dimensional Banach space. Consequently, one can use the existing theory of generalized ordinary differential equations (see e.g. [7,8]) to obtain new results for measure functional differential equations with infinite delay. The idea of transforming a classical functional differential equation to a generalized ordinary differential equation first appeared in the papers [9,10] by C. Imaz, F. Oliva, and Z. Vorel. Later, it was extended to impulsive functional differential equations in the paper [11] by M. Federson and Š. Schwabik, and to measure functional differential equations with finite delay in the paper [1] by M. Federson, J. G. Mesquita and A. Slavík. In [12–14], the correspondence between functional differential equations and generalized ordinary equations was used to obtain various results on boundedness and stability of solutions.

2. Phase space description

In general, solutions of measure functional differential equations are not continuous, but merely regulated functions; recall that a function $f : [a, b] \to \mathbb{R}^n$ is called regulated, if the limits

$$\lim_{s \to t-} f(s) = f(t-) \in \mathbb{R}^n, \quad t \in (a, b] \quad \text{and} \quad \lim_{s \to t+} f(s) = f(t+) \in \mathbb{R}^n, \quad t \in [a, b)$$

exist. Regulated functions on open or half-open intervals are defined in a similar way. Given an interval $I \subset \mathbb{R}$ and a set $B \subset \mathbb{R}^n$, we use the symbol $G(I, B)$ to denote the set of all regulated functions $f : I \to B$, and the symbol $C(I, B)$ to denote the set of all continuous functions $f : I \to B$.

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