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Positive solutions to a class of second-order semilinear elliptic equations in an exterior domain

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ABSTRACT

In this paper, by using the Schauder–Tikhonov fixed point theorem, we investigate the existence of positive solutions to a convection-diffusion equation $\Delta u + f(x, u, \nabla u) = 0$, in an exterior domain of $\mathbb{R}^n$.

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1. Introduction

In this paper, we study the positive solutions to the following semilinear elliptic equation [1]:

$$\Delta u + f(x, u, \nabla u) = 0, \quad x \in \Omega_A, \tag{1.1}$$

where $\Omega_A = \{x \in \mathbb{R}^n, |x| > A\}$, with $n \geq 3$ and $A \geq 1$, while f is a locally Hölder continuous function in $\Omega_A \times \mathbb{R} \times \mathbb{R}^n$. Eq. (1.1) can be regarded as a mathematical model for many physical, chemical and biological phenomena. As a special case, we know that a class of semilinear equations

$$\Delta u + f(x, u) + g(|x|)x \cdot \nabla u = 0, \quad x \in \Omega_A$$

has received numerous investigations in the last a few years [2–4]. Related results also have been established in [5–9].

To prove that the Eq. (1.1) exists positive solutions, we usually consider the comparison second-order differential equation

$$y''(x) + F(x, y(x), y'(x)) = 0, \quad x \in [a, \infty), \tag{1.2}$$

where $a \geq 0$, $F : [a, \infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous in all three variables.

In [6] a criterion is established for the existence of positive solutions to Eq. (1.2) by Mats Ehrnström as follows: let $k : [a, \infty) \to [0, \infty)$ be a continuous function, such that, for any $u, u', v, v' \in \mathbb{R}$ and $x \in [a, \infty)$,

$$|F(x, u, u') - F(x, v, v')| \leq k(x)(|u - v| + |u' - v'|), \tag{1.3}$$

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with

$$\int_a^\infty tk(t)dt < \infty. \tag{1.4}$$

In addition, suppose that for nonnegative u and u', the function $F(x, u, u')$ is nonnegative. Let $M > 0$, if we have that

$$\int_a^\infty tF(t, u(t), u'(t))dt \le M, \tag{1.5}$$

then there exists a unique positive solution to Eq. (1.2), with

$$\lim_{t\to\infty} y(x) = M.$$

In this paper, in order to establish the existence of positive solutions to Eq. (1.1), we consider the comparison second-order differential equation

$$h''(s) + a(s)g\left(b(s), \frac{h(s)}{s}, \left(\frac{h(s)}{s}\right)'\right) = 0, \tag{1.6}$$

where

$$a(s),\ b(s) \in C\left(\mathbb{R}, (0, \infty)\right), \quad g : [c, \infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}.$$

We present a criterion without conditions (1.3) and (1.4). We use the supersolution-subsolution method and the Schauder–Tikhonov fixed point theory, which are based on a work of Mats Ehrnström (see [6]).

We now conclude this introduction by outlining the rest of this paper. In Section 2, we will present some definitions and results, which will be used in this paper. In Section 3, we will list main results and give proofs.

2. Preliminaries

Definition 2.1. We say that $u \in C^2(\Omega_A, \mathbb{R})$ is a solution to Eq. (1.1), if $\Delta u + f(x, u, \nabla u) = 0$ for $x \in \Omega_A$, $u \in C^2(\Omega_A, \mathbb{R})$ is a subsolution if $\Delta u + f(x, u, \nabla u) \ge 0$ for $x \in \Omega_A$, whereas for supersolution the opposite inequality has to hold throughout Ω_A.

Lemma 2.1 ([10]). *Assume that $f : \Omega_A \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is Hölder continuous. If there exists a nonnegative subsolution $w(x)$ and a positive supersolution $v(x)$ to Eq. (1.1) in Ω_A, such that $w(x) \le v(x)$ for all $x \in \overline{\Omega_A}$, then the Eq. (1.1) has a solution $u(x)$ in Ω_A, such that $w(x) \le u(x) \le v(x)$ in $\overline{\Omega_A}$, In particular $u(x) = v(x)$ on S_A.*

Lemma 2.2 (*The Schauder-Tikhonov Fixed Point Theorem [11]*). *Let X be a Banach space and $K \subset X$ be a nonempty, closed, bounded and convex set. If the operator $T : K \to X$ continuously maps K into itself and $T(K)$ is relatively compact in X, then T has a fixed point $x \in K$.*

We assume that

$$X = \left\{ v \in C^1(\mathbb{R}_+, \mathbb{R}) : \sup_{t\ge 0} |v(t)| < \infty,\ \ \sup_{t\ge 0} |v'(t)| < \infty \right\}.$$

Proposition 2.3 ([12]). *. The space X is Banach space under the norm*

$$\|v\| = \max\left\{ \sup_{t\ge 0} |v(t)|,\ \ \sup_{t\ge 0} |v(t) + tv'(t)| \right\}.$$

3. Main results

Before going to the main theorem, we investigate Eq. (1.6) first.
Let $M > 0$,

$$K = \{v \in X : 0 \le v(t) \le M,\ 0 \le v(t) + tv'(t) \le M\}, \quad \text{with } v(s) = \frac{h(s)}{s}.$$

Then we have:

Theorem 3.1. *Let $M > 0$. If for any $v \in K$, we have*

$$\int_c^\infty sa(s)g\left(b(s), v(s), v'(s)\right) ds \le M, \tag{3.1}$$

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