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The domain of attraction and the stability region for stochastic partial differential equations with delays

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ABSTRACT

In this paper, a stochastic partial differential equation with delays is considered. On the basis of the properties of nonnegative matrices, stochastic convolution and the inequality technique, sufficient conditions for determining the domain of pth-moment attraction and the pth-moment asymptotic stability region are obtained. An example is also discussed, to illustrate the efficiency of the results obtained.

1. Introduction

The qualitative properties of solutions of differential equations have been investigated by many authors (see, for example, [1–5]). Most of the stability results obtained so far are valid either globally in the entire state space or locally in the neighborhood of the equilibrium state. Both kinds of results, as pointed out by Siljak [6, p. 132], are unsatisfactory in a certain sense. As we know, the equilibrium point for a general nonlinear system may not be unique. Even if the nonlinear system has a unique equilibrium point, the equilibrium point for the corresponding stochastic nonlinear system may not be unique because of the stochastic effect. Therefore, the global results are unrealistic. On the other hand, local results are unsatisfactory in that it is not certain how far initial conditions can be allowed to vary without disrupting the stability properties established in the immediate vicinity of an equilibrium state. A compromise between these two extremes is provided by estimates of the stability region and the domain of attraction of dynamical systems. These topics have attracted research interest from many authors and various results are reported. Siljak [6] studied the stability region of ordinary differential equations by using a Lyapunov function. Lakshmikantham and Leela [7] estimated the actual stability region of ordinary differential equations by using differential inequalities. Kolmanovskii and Nosov [1] discussed the domain of attraction of functional differential equations by using two Lyapunov functions. Xu et al. [8] dealt with the domain of attraction of nonlinear functional difference equations on the basis of a difference inequality. Xu et al. [9] considered the invariant set and the stability region of partial differential equations with time delay by virtue of the properties of nonnegative matrices and the inequality technique. Li et al. [10] investigated the asymptotic and exponential stability region for a class of nonlinear integro-differential equations. Xu and Xu [11] considered the attracting and invariant sets for a class of impulsive stochastic functional differential equations.

Many real world problems in science and engineering can be modeled by using nonlinear stochastic partial differential equations. The existence, uniqueness and asymptotic behavior of solutions of the stochastic partial differential equations

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have been considered by many authors (see [12–21] and references therein). However, the problem of determining the domain of attraction and the stability region for nonlinear stochastic partial differential equations with delays is more complicated and still open. Hence, techniques and methods should be developed and explored.

Taking motivation from the above, our main aim in this paper is to study the domain of attraction and the stability region for nonlinear stochastic partial differential equations with delays. On the basis of the properties of nonnegative matrices, stochastic convolution and the inequality technique, conditions for determining the domain of pth-moment attraction and the pth-moment asymptotic stability region, $p \geq 2$, are obtained. An example is also discussed, to illustrate the efficiency of the results obtained.

2. Preliminaries

In this section, we introduce some notation and recall some basic definitions.

Let I denote an $n \times n$ unit matrix. R^n denotes the n-dimensional Euclidean space and R^n_+ denotes the n-dimensional nonnegative Euclidean space. For a nonnegative matrix $M \in R^{n\times n}$, let $\rho(M)$ be the spectral radius of M.

Let U and H be separable Hilbert spaces and let $\mathcal{L}(U, H)$ be the space of all bounded linear operators from U to H. We denote the norms of elements in U, H and $\mathcal{L}(U, H)$ by symbols $|\cdot|_U, |\cdot|_H$ and $|\cdot|_{\mathcal{L}(U,H)}$, respectively.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in R}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\in R}$ satisfying the usual conditions. We are given a Q-Wiener process in the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in R}, \mathbb{P})$ and having values in U, i.e. $W(t)$ is defined as (see [22])

$$W(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\omega_n(t)e_n, \quad t \geq 0,$$

where $\omega_n(t)$ $(n = 1, 2, 3, \ldots)$ is a sequence of real-valued one-dimensional standard Brownian motions, mutually independent, on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in R}, \mathbb{P})$; $\lambda_n \geq 0$ $(n = 1, 2, 3, \ldots)$ are nonnegative real numbers such that $\sum_{n\geq 1} \lambda_n < \infty$; $\{e_n\}_{n\geq 1}$ is a complete orthonormal basis in U, and $Q \in \mathcal{L}(U, U)$ is the incremental covariance operator of the process $W(t)$, which is a symmetric nonnegative trace class operator defined by

$$Qe_n = \lambda_n e_n, \quad n = 1, 2, 3, \ldots.$$

Define $U_0 = Q^{\frac{1}{2}}U$, and let L_2^0 be the space of all Hilbert–Schmidt operators $L_2^0 = L_2(U_0, H)$ from U_0 to H. The space L_2^0 is also a separable Hilbert space equipped with the following norm:

$$|\Psi|^2_{L_2^0} = \mathrm{Tr}\,(\Psi Q \Psi^*).$$

Let $H^n = \underbrace{H \times \cdots \times H}_{n}$, and $C(J, H^n)$ denote the space of all continuous functions from the interval J into H^n equipped with a supremum norm. Let us fix a $\tau > 0$ and consider $c > 0$. If we have a function $u(t) \in C([-\tau, c], H^n)$, for each $t \in [0, c]$ we denote by $u_t \in C([-\tau, 0], H^n)$ the function defined by $u_t(s) = u(t+s)$, $-\tau \leq s \leq 0$. In particular, let $C_{H^n} \triangleq C([-\tau, 0], H^n)$.

For a separable Banach space V, let $L^p(\Omega, V)$ be the space of $\mathcal{F}_t$-measurable, V-valued stochastic processes which are L^p in the Bochner sense, $p \geq 2$. In particular, for $L^p(\Omega, H^n)$ and $L^p(\Omega, C_{H^n})$, denote their norms by $\|\cdot\|_1$ and $\|\cdot\|_2$, respectively, where

$$\|\psi\|_1 = \left[\mathbb{E}\sum_{i=1}^{n} |\psi_i(\omega)|^p_H\right]^{\frac{1}{p}}, \qquad \|\varphi\|_2 = \left[\mathbb{E}\sup_{-\tau\leq\sigma\leq 0}\sum_{i=1}^{n} |\varphi_i(\sigma, \omega)|^p_H\right]^{\frac{1}{p}},$$

for any $\psi = (\psi_1, \ldots, \psi_n)^T \in L^p(\Omega, H^n)$, $\varphi = (\varphi_1, \ldots, \varphi_n)^T \in L^p(\Omega, C_{H^n})$.

For any $p \geq 2$, $\psi \in H^n$ or $\varphi \in C_{H^n}$, we define

$$[\psi]^p = (|\psi_1|^p_H, \ldots, |\psi_n|^p_H)^T, \qquad [\varphi]^p_\tau = (|\varphi_1|^p_\tau, \ldots, |\varphi_n|^p_\tau)^T,$$

where $|\varphi_i|^p_\tau = \sup_{-\tau\leq\theta\leq 0} |\varphi_i(\theta)|^p_H$, $i \in \mathcal{N} \triangleq \{1, 2, \ldots, n\}$.

In this paper, we will study the following stochastic partial differential equation:

$$\begin{cases} \mathrm{d}u_i(t) = [A_i u_i(t) + f_i(t, u_t)]\mathrm{d}t + g_i(t, u_t)\mathrm{d}W(t), & t \geq t_0, \\ u_{i,t_0}(s) = \xi_i(s), \quad s \in [-\tau, 0],\ i \in \mathcal{N}, \end{cases} \tag{1}$$

where for each $i \in \mathcal{N}$, A_i is the infinitesimal generator of a semigroup of bounded linear operators $S_i(t)$, $t \geq 0$ in H, $f_i : R \times C_{H^n} \to H$ and $g_i : R \times C_{H^n} \to L_2^0$ are continuous nonlinear mappings, $W(t)$ is a Q-Wiener process defined above, and $\xi = (\xi_1, \ldots, \xi_n)^T \in L^p(\Omega, C_{H^n})$ is the initial function. Denote by $u(t) = (u_1, \ldots, u_n)^T$ or $u(t; t_0, \xi)$ the solution of system (1) with the initial data ξ.