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Solvability and continuous dependence results for second order nonlinear evolution inclusions with a Volterra-type operator[☆]

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ABSTRACT

The paper deals with second order nonlinear evolution inclusions and their applications. We study evolution inclusions involving a Volterra-type integral operator, which are considered within the framework of an evolution triple of spaces. First, we deliver a result on the unique solvability of the Cauchy problem for the inclusion by combining a surjectivity result for multivalued pseudomonotone operators and the Banach contraction principle. Next, we provide a theorem on the continuous dependence of the solution to the inclusion with respect to the operators involved in the problem. Finally, we consider a dynamic frictional contact problem of viscoelasticity for materials with long memory and indicate how the result on evolution inclusion is applicable to the model of the contact problem.

1. Introduction

The purpose of this paper is to study the following second order evolution inclusion involving a Volterra-type integral operator

$$u''(t) + A(t, u'(t)) + B(t, u(t)) + \int_0^t C(t-s)u(s)\,ds + F(t, u(t), u'(t)) \ni f(t)$$

which is considered on a finite time interval $(0, T)$ in the framework of an evolution triple of spaces (V, H, V^*). We provide conditions on a unique solvability of the inclusion and on the continuous dependence of the solution to the inclusion with respect to the operators involved in the problem. Throughout the paper, we take a special interest in the multivalued

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term F for which we employ hypotheses that allow for applications to inclusions with multifunctions generated by the Clarke subdifferential of a locally Lipschitz function. The motivation for our study comes from the theory of hemivariational inequalities which is closely related to the theory of inclusions of subdifferential type.

The second order evolution inclusions have been studied earlier by Ahmed and Kerbal [1], Bian [2], Denkowski et al. [3], Kulig [4], Migórski [5,6], Migórski et al. [7], Papageorgiou [8], Papageorgiou and Yannakakis [9,10] and Tolstonogov [11]. We underline that, except [4,7], none of the aforementioned papers on nonlinear evolution inclusions can be applied in our study because of their restrictive hypotheses on the multivalued term which was supposed to have values in a pivot space H. The existence results for inclusions involving multifunctions with values in the space H cannot be applied for hemivariational inequalities. The reason lies in the fact that for the hemivariational inequalities, the associated the multivalued mapping F has values in the space Z^* which is larger than H (cf. hypothesis $H(F)$ in Section 3.1). Moreover, we have employed a method which is different than those of [1–3,5,6,8–11] and which combines a surjectivity result for pseudomonotone operators with the Banach contraction principle (cf. Theorem 1.3.71 of [3,7]). We mention that the result in [1] follows from a standard application of the Galerkin technique and the a priori estimates, in papers [2,5,6] a method based on the Kakutani-Fan fixed point theorem was employed, and [11] investigates the existence results on the basis of the Schauder fixed point theorem and the theory of selectors of multifunctions.

We establish results on global and local unique solvability of the Cauchy problem for the second order evolution inclusion with nonlinear operators and Volterra-type integral term. The inclusion without the Volterra-type integral term and with time-independent operator B has been studied in [3] with $F\colon (0,T)\times H\times H \to 2^H$, Migórski and Ochal [12] in a case B is linear, continuous, symmetric and coercive operator, and in [13,14] in a case B is linear, continuous, symmetric and nonnegative. In this paper we treat the problem with a time-dependent Lipschitz operator $B(t,\cdot)$ and with a linear and continuous kernel operator $C(t)$ in the integral term. We provide a result on the continuous dependence of the solution with respect to the operators A, B and C. It is shown that the sequence of the unique solutions corresponding to perturbed operators A_ε, B_ε and C_ε converges in a suitable sense to the unique solution corresponding to unperturbed operators A, B and C.

We underline that our results are applicable to hemivariational inequalities which are the variational formulations of dynamic contact problems involving nonconvex superpotentials. In the last section of this paper, we provide an example of a mechanical problem for viscoelastic materials with long memory which illustrates such application. We remark that the notion of hemivariational inequality is based on the generalized gradient of Clarke [15] and has been introduced in the early 1980s by Panagiotopoulos [16,17]. We note that the existence of solutions to the second order evolution inclusions as well as to the corresponding dynamic hemivariational inequalities has been studied, for instance, in [4,13,18–22]. Monographs on mathematical theory of hemivariational inequalities include Panagiotopoulos [16,17], Naniewicz and Panagiotopoulos [23], Migórski et al. [7], and we refer the reader there for a wealth of additional information about these and related topics. On the other hand, results on Mathematical Theory of Contact Mechanics can be found in monographs of Han and Sofonea [24], and Shillor et al. [25].

The paper is structured as follows. In Section 2 we recall some preliminary material. In Section 3 we study a class of second order nonlinear evolution inclusions involving a Volterra-type integral operator. For this class we give a result on the existence and uniqueness of solutions to the Cauchy problem for the inclusion under investigation. Section 4 is devoted to the study of the dependence of the solution to the evolution inclusion on the operators involved in the problem. Section 5 contains a description of a mechanical model to which our results are applicable.

2. Preliminaries

In this section we recall the basic notations needed in the sequel (see e.g. [3,23,26,27]).

Given a Banach space $(X, \|\cdot\|_X)$, the symbol w-X is always used to denote the space X endowed with the weak topology. If $U \subset X$, then we write $\|U\|_X = \sup\{\|x\|_X \mid x \in U\}$. Let (Ω, Σ) be a measure space and $F\colon \Omega \to 2^X$. The multifunction F is called graph measurable if its graph $\operatorname{Gr} F = \{(\omega, x) \in \Omega \times X \mid x \in F(\omega)\}$ belongs to the product of σ-algebra $\Sigma \times \mathcal{B}(X)$, $\mathcal{B}(X)$ being the Borel σ-algebra of X. We use the following notation

$$\mathcal{P}_{f(c)}(X) = \{A \subseteq X \mid A \text{ is nonempty, closed, (convex)}\};$$

$$\mathcal{P}_{(w)k(c)}(X) = \{A \subseteq X \mid A \text{ is nonempty, (weakly) compact, (convex)}\}.$$

The notation $\mathcal{L}(E,F)$ stands for the space of linear continuous operators defined on a Banach space E with values in a Banach space F. Let $T\colon Y \to 2^{Y^*}$ be a multivalued operator, where Y is a reflexive Banach space and let $\langle\cdot,\cdot\rangle$ be the duality pairing for (Y^*, Y). An operator T is said to be *bounded* if for every bounded set $C \subset Y$, the set $T(C)$ is bounded in 2^{Y^*}. It is *upper semicontinuous (usc)* if set $T^-(C) = \{y \in Y \mid Ty \cap C \neq \emptyset\}$ is closed in Y for any closed subset $C \subset Y^*$. It is said to be *coercive* if there exists a function $c\colon \mathbb{R}_+ \to \mathbb{R}$ with $\lim_{r\to+\infty} c(r) = +\infty$ such that for all $y \in Y$ and $y^* \in Ty$, we have $\langle y^*, y\rangle \geq c(\|y\|)\|y\|$. An operator T is said to be *pseudomonotone* if it satisfies

(a) for every $y \in Y$, Ty is a nonempty, convex and weakly compact set in Y^*;
(b) T is usc from every finite dimensional subspace of Y into Y^* endowed with the weak topology; and
(c) if $y_n \to y$ weakly in Y, $y_n^* \in Ty_n$ and $\limsup\langle y_n^*, y_n - y\rangle \leq 0$, then for each $z \in Y$ there exists $y^*(z) \in Ty$ such that $\langle y^*(z), y - z\rangle \leq \liminf\langle y_n^*, y_n - z\rangle$.