Dynamics of multi-species competition–predator system with impulsive perturbations and Holling type III functional responses

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A R T I C L E   I N F O

A B S T R A C T

We investigate the dynamics of a class of multi-species predator–prey interaction models with Holling type III functional responses based on systems of nonautonomous differential equations with impulsive perturbations. Sufficient conditions for existence of a positive periodic solution are investigated by using a continuation theorem in coincidence degree theory, which have been extensively applied in studying existence problems in differential equations and difference equations. In addition, sufficient criteria are established for the global stability and the globally exponential stability of the system by using the comparison principle and the Lyapunov method.

1. Introduction

Ecological predator–prey systems have been studied extensively by many authors [1–8]. In the real world, any biological or environmental parameters are naturally subject to fluctuation in time, so it is reasonable to study the corresponding nonautonomous system. Considering the biological and environmental periodicity (e.g. seasonal effects of weather, food supplies, mating habits), we focus on the existence of a periodic solution with strictly positive components of (1.1).

Differential equations with impulsive effects form a wide set of different problems. During the last three decades those problems were intensively studied. Some authors devote themselves to the study of impulsive differential equations [9–13]. The main definitions and results of the theory of systems of ordinary differential equations with impulse effects were given in [9]. Similarity and differentiality of such problems of applied mathematics with corresponding problems of ordinary differential equations (and without the conditions of impulsive effects) were demonstrated and general characteristics of these systems were described. Periodic and almost-periodic solutions of differential equations with impulsive effects were studied in [14].

Some impulsive factors have also great impact on the growth of a population. For example, we notice that the births of many species are not continuous but happen at some regular time. (For instance, the births of some wildlife are seasonal.) Moreover, human beings have been harvesting or stocking species at some time, then the species is affected by another type of impulse. If we incorporate these impulsive factors into the models of population interactions, the models must be governed by impulsive ordinary differential equations. The first author investigated the existence of positive periodic solutions of a nonautonomous competitive Lotka–Volterra system with impulse and Beddington–DeAngelis functional

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response [15]. An N-dimensional Lotka–Volterra system with fixed moments of impulsive perturbations is given in [16] by Ahmad and Stamova. By means of piecewise continuous functions which are modifications of classical Lyapunov functions they give sufficient conditions for asymptotic stability of the solutions. Inspired by [15,16], in this paper, we focus our attention on the existence of a periodic solution and globally asymptotic stability of solutions for a multi-species competition–predator system with impulses in which the competitions among predator species and among prey species are simultaneously considered. Some authors have been concerned with the Holling functional response in the study of ecological systems [3,17–19], the Holling type III functional response occurs in predators which increase their search activity with increasing prey density. For example, many predators respond to kairomones (chemicals emitted by prey) and increase their activity. Polyphagous vertebrate predators (e.g., birds) can switch to the most abundant prey species by learning to recognize it visually. Mortality first increases with prey increasing density, and then declines. Thus we also consider the Holling type III functional response in predators. The primary approach is based on the coincidence degree and its related continuation theorem [20], which has been widely used in dealing with the existence of periodic solutions of differential equations. We will investigate the following nonautonomous predator–prey system

$$
\begin{cases}
\dot{x}_i(t) = x_i(t)\left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t) - \sum_{k=n+1}^{n+m} \dfrac{c_{ik}(t)x_i(t)y_k(t)}{f_{ik}(t) + x_i^2(t)} \right], \\[2ex]
\dot{y}_j(t) = y_j(t)\left[-r_j(t) + \sum_{k=1}^{n} \dfrac{d_{jk}(t)x_k^2(t)}{f_{jk}(t) + x_k^2(t)} - \sum_{k=n+1}^{n+m} \delta_{jk}(t)y_k(t) \right], \\[2ex]
\quad t \neq t_k (k \in \mathbb{N}^+),\ i = 1, \ldots, n,\ j = n+1, \ldots, n+m, \\
\Delta x_i(t) = x_i(t^+) - x_i(t^-) = (b_{ik} + h_{ik})x_i(t), \\
\Delta y_j(t) = y_j(t^+) - y_j(t^-) = (b_{jk} + h_{jk})y_j(t), \quad t = t_k (k \in \mathbb{N}^+),\ i = 1, \ldots, n,\ j = n+1, \ldots, n+m,
\end{cases}
\tag{1.1}
$$

where

- $x_i(t)$ $(i = 1, 2, \ldots, n)$ denote the densities of prey species at time t, respectively;
- $y_j(t)$ $(j = n+1, n+2, \ldots, n+m)$ denote the density of predator species at time t, respectively;
- b_{ik} and b_{jk} represent the birth rate of $x_i(t)$ and $y_j(t)$ at time t, respectively;
- h_{ik} and h_{jk} represent the harvesting (stocking) rate of $y_j(t)$ at time t, respectively. When $h_{ik}, h_{jk} > 0$, it stands for harvesting, while $h_{ik}, h_{jk} < 0$ means stocking;
- $x_i(t^+)$ and $x_i(t^-)$ represent the right and left limits of $x_i(t)$ at t, $y_j(t^+)$ and $y_j(t^-)$ represent the right and left limits of $y_j(t)$ at t.

   Let $g(t)$ be a bounded continuous function on $\mathbb{R}$. Define

$$
g^{\ell} = \inf_{t \in \mathbb{R}} g(t), \qquad g^u = \sup_{t \in \mathbb{R}} g(t).
$$

Particularly, if $g(t)$ are T-periodic functions with respect to t, then

$$
\bar{g} = \frac{1}{T} \int_0^T f(t)\mathrm{d}t.
$$

The ranges of the indices $i \in \{1, 2, \ldots, n\}$ and $j \in \{n+1, \ldots, n+m\}$ are used in this paper unless otherwise stated.
   Throughout the paper, we give the hypothesis as follows.

(A1) For any $t \in \mathbb{R}$, $b_i(t), a_{ik}(t), r_j(t), d_{jk}(t), f_{jk}$ $(k = 1, \ldots, n)$, $\delta_{jk}(t), c_{ik}(t), f_{ik}$ $(k = n+1, n+2, \ldots, n+m)$, are non-negative continuous T-periodic functions and $f_{ij} = f_{ji}$.

(A2) $b_{ik}, b_{jk} > 0$, $1 + b_{ik} + h_{ik} > 0$, $1 + b_{jk} + h_{jk} > 0$, $b_{ik}, h_{ik}, b_{jk}, h_{jk}(k \in \mathbb{N}^+)$ are constants. There exists a positive integer q, such that $t_{k+q} = t_k + T$, $b_{i(k+q)} = b_{ik}$, $b_{j(k+q)} = b_{jk}$, $h_{i(k+q)} = h_{ik}$, $h_{j(k+q)} = h_{jk}(k \in \mathbb{N}^+)$. Without loss of generality, we also suppose that $t_k \neq 0$ and $[0, T] \cap \{t_k | k \in \mathbb{N}^+\} = \{t_1, t_2, \ldots, t_s\}$, then it follows that $q = s$.

(A3) $x_i(t), y_j(t)$ is left-continuous at t_k, i.e., the following relations are satisfied:

$$
\begin{aligned}
x_i(t_k^-) &= x_i(t_k), & x_i(t_k^+) &= (1 + b_{ik} + h_{ik})x_i(t_k), & k \in \mathbb{N}^+, \\
y_j(t_k^-) &= y_j(t_k), & y_j(t_k^+) &= (1 + b_{jk} + h_{jk})y_j(t_k), & k \in \mathbb{N}^+.
\end{aligned}
$$

(A4) $t_1 < t_2 < \cdots$ and $\lim_{k \to \infty} t_k = \infty$.

(A5) $x_i(t_0^+) > 0$, $y_j(t_0^+) > 0$.

   A brief description of the organization of this paper is as follows. The basic concepts and lemmas are given in Section 2. The main results in this paper are stated in Theorems 3.1, 4.5 and 4.6.