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Spin torque oscillator neuroanalog of von Neumann's microwave computer☆

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ABSTRACT

Frequency and phase of neural activity play important roles in the behaving brain. The emerging understanding of these roles has been informed by the design of analog devices that have been important to neuroscience, among them the neuroanalog computer developed by O. Schmitt and A. Hodgkin in the 1930s. Later J. von Neumann, in a search for high performance computing using microwaves, invented a logic machine based on crystal diodes that can perform logic functions including binary arithmetic. Described here is an embodiment of his machine using nano-magnetics. Electrical currents through point contacts on a ferromagnetic thin film can create oscillations in the magnetization of the film. Under natural conditions these properties of a ferromagnetic thin film may be described by a nonlinear Schrödinger equation for the film's magnetization. Radiating solutions of this system are referred to as spin waves, and communication within the film may be by spin waves or by directed graphs of electrical connections. It is shown here how to formulate a STO logic machine, and by computer simulation how this machine can perform several computations simultaneously using multiplexing of inputs, that this system can evaluate iterated logic functions, and that spin waves may communicate frequency, phase and binary information. Neural tissue and the Schmitt-Hodgkin, von Neumann and STO devices share a common bifurcation structure, although these systems operate on vastly different space and time scales; namely, all may exhibit Andronov-Hopf bifurcations. This suggests that neural circuits may be capable of the computational functionality as described by von Neumann.

The importance of frequency and phase in neuroscience has been acknowledged in many studies (e.g., see Izhikevich et al., 2003; Nunez, 1995; Izhikevich, 2007; Encyclopedia, 2015), for further references). Models for observed or speculated neural phenomena are often formulated in terms of nonlinear oscillators that describe electromagnetic, biochemical or mechanical processes, including the neuroanalog computer of O. Schmitt and A. Hodgkin in the 1930s, the van der Pol discharge tube models, the Hodgkin-Huxley model of nerve membranes and its heuristics, including the FitzHugh-Nagumo and Morris-Lecar models, and various forms of voltage controlled oscillator neuron models (VCON) (Hoppensteadt, 2013). Conversely there have been important contributions from neuroscience to engineering, such as the Schmitt trigger. Mathematics is a common thread in these multi-disciplinary studies. In particular all of these oscillators can exhibit a similar bifurcation structure, as does the spintronic model presented here.

The motivation for this paper is two-fold. First, it describes an embodiment of von Neumann's invention (von Neumann, 1957) based on our work on spin-torque systems (Maciá et al., 2011), and it demonstrates some of its functionality. Second, it makes a connection between spintronics and brain science through a common bifurcation structure shared by STO and the models from neuroscience listed earlier. While the brain operates on space scales of millimeters and time scales of milliseconds, STO arrays operate on space scales of nanometers and time scales of picoseconds. In particular, such arrays may provide brain-like functions on much smaller and faster scales than those in a behaving brain. However, this article is about physical devices that have been studied in Maciá et al. (2011) and elsewhere, and it is not about the brain. It is not clear how the results here might be interpreted in the context of the brain, nor how magnetic behavior in the brain might occur or what its functions might be. But, there are magnetic elements in the

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brain, so spin waves or their kin may be present. The spin torque phenomena discussed here have been discovered and developed by physicists only recently (see references to this development in Maciá et al. (2011)), and only in 2015 have spin waves been observed (Bonetti et al., n.d.).

There have been many applications and extensions of von Neumann's approach to embedding binary information in the phase of signals using parametrons, MEMS, lasers, Josephson junctions and STO, for example by Goto (1955), Wigington (1959), Mahboob (2008) and Roychowdhury (2014). The novelty here is in demonstrating through simulation that a single STO may perform several logic computations in parallel through multiplexing, that iterated logic statements may be evaluated by an array of STO, and that aggregations of them may transmit and process digital information by means of spin waves.

Section 1 describes von Neumann's invention (von Neumann, 1957) showing how binary arithmetic might be performed by microwave oscillator systems. Section 2 presents simulations showing a single STO can perform simple logic computations, how an array of them can evaluate iterated logic functions, how a single one can perform several separate computations simultaneously by multiplexing of inputs, and how spin waves may transmit and process information about amplitude, frequency and phase. Other applications of this methodology, not shown here, are to switching and control.

As shown here, parametrically driven oscillator systems containing an Andronov-Hopf bifurcation, including STO and those from neuroscience, may perform logic computations and processing of information. Since bursts of action potentials are known to carry amplitude, phase and frequency information (e.g., see Izhikevich et al., 2003; Nunez, 1995), it is possible to speculate that neural circuits may use bursts or other action potential phenomena as mechanisms for performing and communicating logic computations on the space and time scales of neural activity.

The Andronov-Hopf (AH) bifurcation structure is important for the phenomena described here since it appears naturally in Landau-Lifshitz equations. Recall that a canonical model for an AH bifurcation is the ordinary differential equation

$$\frac{dz}{dt} = \jmath\omega z + (\Lambda - |z|^2)z \tag{1}$$

for a complex function $z(t)$ where ω is the center frequency, Λ is the amplification rate and $\jmath^2 = -1$. If $\Lambda < 0$, then $|z| \to 0$ as $t \to \infty$. If $\Lambda > 0$ and if $z(0) \neq 0$, then z approaches an oscillation; namely, $z \to \sqrt{\Lambda}\exp(\jmath\omega t + \phi)$ as $t \to \infty$ where the phase deviation ϕ is determined by z's initial value. The value $\Lambda = 0$ is the bifurcation point for this system. In our case, we replace Λ by $\lambda + C(t)$ where $\lambda < 0$ and C represents external parametric forcing, so when $C(t) > -\lambda$, $|u|$ grows until $C(t)$ returns to $C(t) < -\lambda$. In particular, the system exhibits bifurcation behavior at these junctures. Other bifurcation structures, such as the saddle-node on invariant circle bifurcation (SNIC), may exhibit similar functionality, but we focus on AH here. Parametric forcing of oscillators has been extensively studied in the nonlinear oscillator, physics, neuroscience and engineering literatures.

1. Formal logic and binary arithmetic

The basis of arithmetic in computers is performing addition of binary numbers bit by bit, while keeping track of carry-over. Binary addition may be accomplished using logic gates that are electronic embodiments of the basic operations from formal mathematical logic of OR, AND and NAND. These may be combined to perform all arithmetic operations using NAND logic. In the following, the symbols a and b may be used as generic logical entities; they may be binary digits, sets, signals, or represent other relevant concepts.

Disjunction is calculating a OR b, which is written as $a \vee b$. Conjunction is calculating a AND b, which is written as $a \wedge b$. NAND calculates the negation of a AND b, which is written as $a \barwedge b = \neg(a \wedge b)$, where $\neg$ denotes negation. Note that in NAND logic, the logic function NOT may be calculated using the fact that $\neg a = a \barwedge a$. All of these operations are defined by truth tables.

The sum of the two binary digits a, b (i.e., 0 or 1), may be accomplished using the Exclusive OR logic function, XOR(a, b), which may be expressed in NAND logic. This is discussed in Section 2.2.

1.1. Binary digits as continuous waveforms and truth tables

The digit 1 is represented here by the signal

$$p(\omega_p t) \equiv A_p \cos(2\pi\omega_p t + \phi) \tag{2}$$

for some given amplitude A_p, phase deviation ϕ and frequency ω_p, and the digit 0 is represented by the signal $n(\omega_p t) \equiv A_p \cos(2\pi\omega_p t + \phi \pm \pi) = -p(\omega_p t)$. von Neumann (1957) showed how logic statements may be calculated using various superpositions of these signals along with nonlinear circuitry.

Encoding a binary digit a in terms of phase variables is done using the formulas

$$A = -\cos a\pi \text{ and } a = 1 - \arccos(A)/\pi,$$

where $A = +1$ if $a = 1$ and $A = -1$ if $a = 0$. Then, for example, if $A = 1$, $AA_p\cos(\omega_p t) = p(\omega_p t)$, which represents the binary digit 1, and if $A = -1$, $AA_p\cos(\omega_p t) = n(\omega_p t)$, which represents the binary digit 0.

We write $\mathrm{L}(A, B)$ to represent a logic function operating on binary digits (a, b) where $A = -\cos a\pi$, $B = -\cos b\pi$. In particular,

$$\begin{aligned}
a \vee b: \quad & \mathrm{L}_\vee(A, B) = \mathrm{sign}(1 + A + B)\\
a \wedge b: \quad & \mathrm{L}_\wedge(A, B) = \mathrm{sign}(-1 + A + B)\\
a \barwedge b: \quad & \mathrm{L}_{\barwedge}(A, B) = \mathrm{sign}(1 - 2A - 2B).
\end{aligned}$$

Since there is an odd number of terms in each case, these nonlinear functions have the value ± 1, and they produce the correct truth tables for OR, AND, and NAND:

a	b	$a \vee b$	$a \wedge b$	$a \bar{\wedge} b$	A	B	$L_\vee$	$L_\wedge$	$L_{\barwedge}$
0	0	0	0	1	−1	−1	−1	−1	+1
1	0	1	0	1	+1	−1	+1	−1	+1
0	1	1	0	1	−1	+1	+1	−1	+1
1	1	1	1	0	+1	+1	+1	+1	−1

1.2. Computation of logic functions

Disjunction involves combining two inputs $\delta_1(t), \delta_2(t)$, where the possible inputs are $\delta_j(t) = p(\omega_p t)$ or $n(\omega_p t)$ for $j = 1, 2$. The function

$$p(\omega_p t)L_\vee(A, B)$$

gives the equivalent of $\delta_1(t) \vee \delta_2(t)$. The output is proportional to p if and only if at least one of the input digits is 1.

Conjunction is similar. It combines two inputs $\delta_1(t), \delta_2(t)$ as

$$p(\omega_p t)L_\wedge(A, B),$$

which gives the equivalent of $\delta_1(t) \wedge \delta_2(t)$. The output is proportional to p if and only if both the input digits are 1.

NAND combines the inputs as

$$p(\omega_p t)L_{\barwedge}(A, B),$$

which gives the equivalent of $\delta_1(t) \barwedge \delta_2(t)$. The output is proportional to p if and only if at least one of the input digits is 0.