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Bifurcation of limit cycles by perturbing a piecewise linear Hamiltonian system with a homoclinic loop[☆]

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ABSTRACT

In this paper, we study limit cycle bifurcations for a kind of non-smooth polynomial differential systems by perturbing a piecewise linear Hamiltonian system with the center at the origin and a homoclinic loop around the origin. By using the first Melnikov function of piecewise near-Hamiltonian systems, we give lower bounds of the maximal number of limit cycles in Hopf and homoclinic bifurcations, and derive an upper bound of the number of limit cycles that bifurcate from the periodic annulus between the center and the homoclinic loop up to the first order in ε. In the case when the degree of perturbing terms is low, we obtain a precise result on the number of zeros of the first Melnikov function.

1. Introduction and main results

There are many problems in mechanics, electrical engineering and the theory of automatic control which are described by non-smooth systems; see for instance the works of Filippov [1], Andronov et al. [2], Kunze [3] and the references therein. Recently, a good deal of work has been done to study bifurcations in non-smooth systems including Hopf, homoclinic and subharmonic bifurcations. In [4–6], Hopf bifurcation for non-smooth systems was studied by developing new methods for computing Lyapunov constants. The Melnikov method for Hopf and homoclinic bifurcations was extended to non-smooth systems in [7–9]. The method of averaging has also been extended to non-smooth systems in [10]. More results can be found in [11–18]. However, so far there are few papers in the literature studying homoclinic bifurcations inside the class of piecewise polynomial differential systems. In this work, we study this problem by using the first order Melnikov function of piecewise near-Hamiltonian systems.

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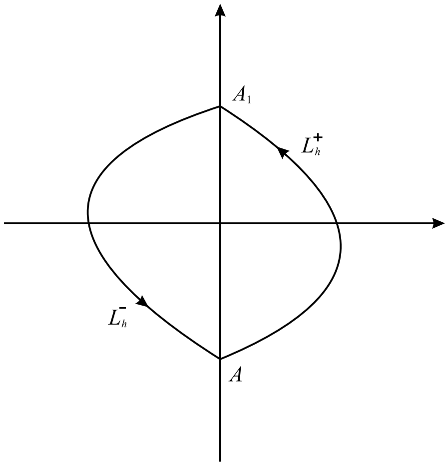

Fig. 1. The closed orbits of (1.1) $|_{\varepsilon=0}$.

In [7], Liu and Han considered the general form of a piecewise near-Hamiltonian system on the plane

$$\begin{cases}\dot{x} = H_y + \varepsilon p(x, y, \delta),\\ \dot{y} = -H_x + \varepsilon q(x, y, \delta),\end{cases} \tag{1.1}$$

where

$$H(x, y) = \begin{cases}H^+(x, y), & x \geq 0,\\ H^-(x, y), & x < 0,\end{cases}$$

$$p(x, y, \delta) = \begin{cases}p^+(x, y, \delta), & x \geq 0,\\ p^-(x, y, \delta), & x < 0,\end{cases}$$

$$q(x, y, \delta) = \begin{cases}q^+(x, y, \delta), & x \geq 0,\\ q^-(x, y, \delta), & x < 0.\end{cases}$$

$H^\pm$, $p^\pm$ and $q^\pm$ are C^∞, $\varepsilon > 0$ is small, $\delta \in D \subset R^m$ is a vector parameter with D compact. This system has two subsystems

$$\begin{cases}\dot{x} = H_y^+ + \varepsilon p^+(x, y, \delta),\\ \dot{y} = -H_x^+ + \varepsilon q^+(x, y, \delta),\end{cases} \tag{1.1a}$$

and

$$\begin{cases}\dot{x} = H_y^- + \varepsilon p^-(x, y, \delta),\\ \dot{y} = -H_x^- + \varepsilon q^-(x, y, \delta),\end{cases} \tag{1.1a}$$

which are called the right subsystem and the left subsystem, respectively. Suppose that (1.1) $|_{\varepsilon=0}$ has a family of periodic orbits around the origin and satisfies the following two assumptions.

Assumption (I). There exist an interval $J = (\alpha, \beta)$, and two points $A(h) = (0, a(h))$ and $A_1(h) = (0, a_1(h))$ such that for $h \in J$

$$H^+(A(h)) = H^+(A_1(h)) = h, \qquad H^-(A(h)) = H^-(A_1(h)) = \widetilde{h}, \qquad a(h) \neq a_1(h).$$

Assumption (II). The subsystem (1.1a) $|_{\varepsilon=0}$ has an orbital arc L_h^+ starting from $A(h)$ and ending at $A_1(h)$ defined by $H^+(x, y) = h, x \geq 0$; the subsystem (1.1b) $|_{\varepsilon=0}$ has an orbital arc L_h^- starting from $A_1(h)$ and ending at $A(h)$ defined by $H^-(x, y) = H^-(A_1(h)), x < 0$.

Under Assumptions (I) and (II), (1.1) $|_{\varepsilon=0}$ has a family of non-smooth periodic orbits $L_h = L_h^+ \cup L_h^-$, $h \in J$. For definiteness, we assume that the orbits L_h for $h \in J$ orientate anticlockwise; see Fig. 1.

By Theorem 1.1 in [7], the first order Melnikov function of system (1.1) has the form

$$M(h, \delta) = \frac{H_y^+(A)}{H_y^-(A)}\left[\frac{H_y^-(A_1)}{H_y^+(A_1)}\int_{L_h^+} q^+dx - p^+dy + \int_{L_h^-} q^-dx - p^-dy\right], \quad h \in J. \tag{1.2}$$