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A semiparametric method for estimating nonlinear autoregressive model with dependent errors

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ABSTRACT

The first-order nonlinear autoregressive model is considered and a semiparametric method is proposed to estimate regression function. In the presented model, dependent errors are defined as first-order autoregressive AR(1). The conditional least squares method is used for parametric estimation and the nonparametric kernel approach is applied to estimate regression adjustment. In this case, some asymptotic behaviors and simulated results for the semiparametric method are presented. Furthermore, the method is applied for the financial data in Iran's Tejarat-Bank.

1. Introduction

This paper is intended to propose the following nonlinear autoregressive model,

$$\begin{aligned} Y_t &= f(Y_{t-1}) + \varepsilon_t, \quad t \in Z \\ \varepsilon_t &= \rho\varepsilon_{t-1} + u_t, \quad |\rho| < 1, \end{aligned} \tag{1.1}$$

where $\{u_t\}$ is a sequence of independent and identically distributed (i.i.d) random variables with zero mean and variance σ^2. Also u_t and Y_t are independent for each t.

In recent years, a combination of parametric forms and nonlinear functions has been used to make a more efficient model in various branches of science, particularly in applied statistics, econometrics and financial studies. Semiparametric single index model, which is known as generalized partial linear time series model [1–5], is a case of this kind of combination. For instance, semiparametric time series models with structure of nonlinear functions were proposed in [3–5]. They considered the following model with nonparametric errors:

$$Y_t = X_t'\beta + U_t \quad \text{with } U_t = g(U_{t-1}) + \varepsilon_t,$$

where X_t is a stationary time series with finite second moment. Here Y_t and U_t are scalars and $g(.)$ is an unknown function. Also $\{U_t\}$ is a stationary time series with zero mean and finite variance. The errors ε_t have the same distributions and are independent random variables. Schick [6] proposed an efficient estimation in a semiparametric additive regression model with autoregressive errors. Truong and Stone [7] considered nonparametric regression models with linear autoregressive error as the following form

$$Y_t = g(X_t) + U_t \quad \text{with } U_t = \theta U_{t-1} + \varepsilon_t,$$

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where (X_t, Y_t) is a stationary bivariate time series and θ is an unknown parameter such that $|\theta| < 1$. As before, $g(.)$ is an unknown function and $\{\varepsilon_t\}$ is a sequence of independent errors with zero mean and finite variance. To estimate the above parameter, they used a semiparametric method.

In financial studies of risk fluctuations models, with inconstant conditionally mean and variance, could not be applied properly since the errors are dependent. See [8,9]. From 1995 to 2005, a partially linear autoregressive conditional heteroskedasticity model, which is somehow with dependent errors, such as

$$Y_t = g(Y_{t-1}) + \beta Y_{t-1} + e_t, \tag{1.2}$$

in econometrics was suggested, where $\{e_t\}$ is assumed to be stationary and $\sigma^2(y) = E[e_t^2|Yt - 1 = y]$ is a smooth function of y. Both β and g are identifiable [9–12]. Zhuoxi et al. [13] successfully applied the following modified version of model (1.2):

$$Y_t = f(Y_{t-1}) + \varepsilon_t.$$

They have estimated $f(.)$ using semiparametric methods where the errors are identity independent variables with zero mean and variance σ^2. Also Y_t and ε_t are independent for each t.

Since mean and variance of errors have periodic variations in financial studies and econometrics time series data, we propose the model (1.1) such that the dependent errors have a first-order autoregressive model.

We suppose that $f(.)$ has a parametric framework, namely a parametric model as

$$f(x) \in \{g(x, \theta); \theta \in \Theta\} \quad \text{or} \quad f(x) \in \left\{g^*(x)h(\theta); \theta \in \Theta\right\}, \tag{1.3}$$

where $\Theta \subseteq R^p$ is a parametric space.

The regression function $f(.)$ is estimated by $\widehat{f}(x) = g(x, \widehat{\theta})$ or $\widehat{f}(x) = g^*(x)h(\widehat{\theta})$ where $\widehat{\theta}$ is an estimator of θ. On the other hand, if little is known about the nature of $f(.)$, a nonparametric approach is desirable [14,15].

For example, a nonparametric estimator based on the Nadaraya–Watson method applied an estimator of the autoregression function $f(.)$, known as a kernel estimator, which is defined as:

$$\widehat{f}(x) = \frac{\sum_{j=1}^{n} K\left(\frac{Y_{j-1}-x}{h_n}\right) Y_j}{\sum_{j=1}^{n} K\left(\frac{Y_{j-1}-x}{h_n}\right)}, \tag{1.4}$$

where $K(.)$ and h_n are kernel and bandwidth respectively. This kernel estimator is a special case of the local polynomial estimator which was proposed by Hardle and Tsybakov [16]. If the parametric assumption (1.3) does not hold, the result of the parametric method can be a deceptive inference about the autoregression function. Zhuoxi et al. [13] suggested semiparametric form $g(x, \theta)\xi(x)$ for the unknown autoregression function. Since $L2$-fitting, the natural consideration of a criterion, is used to estimate the adjustment factor, we can ultimately obtain the following estimator

$$\widehat{f}(x) = g(x, \widehat{\theta})\widehat{\xi}_1(x) \quad \text{or} \quad \widehat{f}(x) = g^*(x)h(\widehat{\theta})\widehat{\xi}_2(x). \tag{1.5}$$

Hence, we use a combination of parametric method and nonparametric adjustment. The parameters and nonparametric adjustment are estimated by using conditional least squares method and smooth-kernel method respectively.

We shortly explain the contents of the manuscript. In Section 2, a least squares estimation is considered to estimate parameters ρ, σ^2 and θ. Also in this section the semiparametric regression estimator is introduced by a natural consideration of the local $L2$-fitting criterion. The asymptotic behaviors of the estimator is investigated in Section 3. In Section 4, a simulation study is performed to confirm the advantage of this method. Finally, this model is used to estimate the financial data in Tejarat-bank of Iran.

2. Semiparametric method with dependent errors

We consider the following model:

$$Y_t = f(Y_{t-1}) + \varepsilon_t \quad t \epsilon Z, \quad \text{with } \varepsilon_t = \rho\varepsilon_{t-1} + u_t.$$

We get

$$\varepsilon_t = Y_t - f(Y_{t-1}) \to \varepsilon_{t-1} = Y_{t-1} - f(Y_{t-2}),$$

and therefore,

$$Y_t = f(Y_{t-1}) + \rho(Y_{t-1} - f(Y_{t-2})) + u_t. \tag{2.1}$$

We want to estimate regression function $f(x)$ that can be formed as $g(x, \theta)$ or $g^*(x)h(\theta)$, where $g(x, \theta)$ or $g^*(x)h(\theta)$ are known functions of x and θ. the true value of θ is denoted by θ_0. The θ_0 is defined as follows:

$$\theta_0 = \underset{\theta\in\Theta,\ |\rho|<1}{\arg\min}\ E(Y_t - E_\theta(Y_t \mid Y_{t-1}) - \rho(Y_{t-2} - E_\theta(Y_t \mid Y_{t-2})))^2.$$