

Bifurcation of limit cycles from a heteroclinic loop with a cusp[☆]

Xianbo Sun^a, Maoan Han^{a,*}, Junmin Yang^b

^a Department of Mathematics, Shanghai Normal University, Shanghai, 200234, China

^b College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang, 050016, China

ARTICLE INFO

Article history:

Received 13 July 2010

Accepted 19 January 2011

Keywords:

Nilpotent cusp
Heteroclinic loop
Melnikov function
Limit cycle
Bifurcation

ABSTRACT

In this article, we study the expansion of the first Melnikov function of a near-Hamiltonian system near a heteroclinic loop with a cusp and a saddle or two cusps, obtaining formulas to compute the first coefficients of the expansion. Then we use the results to study the problem of limit cycle bifurcation for two polynomial systems.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction and main results

Consider a C^∞ plane system of the form

$$\dot{x} = H_y + \varepsilon p(x, y, \delta), \quad \dot{y} = -H_x + \varepsilon q(x, y, \delta) \quad (1.1)$$

where $H(x, y)$, $p(x, y, \delta)$ and $q(x, y, \delta)$ are C^∞ functions in (x, y, δ) with δ a vector parameter varying in a compact set $D \subset \mathbb{R}^n$. For $\varepsilon = 0$ (1.1) becomes

$$\dot{x} = H_y, \quad \dot{y} = -H_x \quad (1.2)$$

which is a Hamiltonian system. Hence, system (1.1) is called a near-Hamiltonian system. Usually we suppose the unperturbed system (1.2) has a family of periodic orbits L_h defined by the equation $H(x, y) = h$. The boundary of the family $\{L_h\}$ can be a center or a homoclinic or a heteroclinic loop. An important topic is to study the number of limit cycles of the perturbed system in a neighborhood of a center, a homoclinic or a heteroclinic loop with either saddles or cusps. In this respect, a Melnikov function of the form

$$M(h, \delta) = \oint_{L_h} q dx - p dy \quad (1.3)$$

plays an important role; see [1–3].

Let a boundary of the family $\{L_h\}$ be a closed curve having at most two singular points. Then we have the following possibilities for the curve.

- (1) It is a homoclinic loop with one hyperbolic saddle.
- (2) It is a homoclinic loop with one cusp.
- (3) It is a heteroclinic loop having 2 heteroclinic orbits connecting 2 hyperbolic saddles.

[☆] The project was supported by the National Natural Science Foundation of China (10971139).

* Corresponding author. Tel.: +86 21 64323580; fax: +86 21 64328672.

E-mail address: mahan@shnu.edu.cn (M. Han).

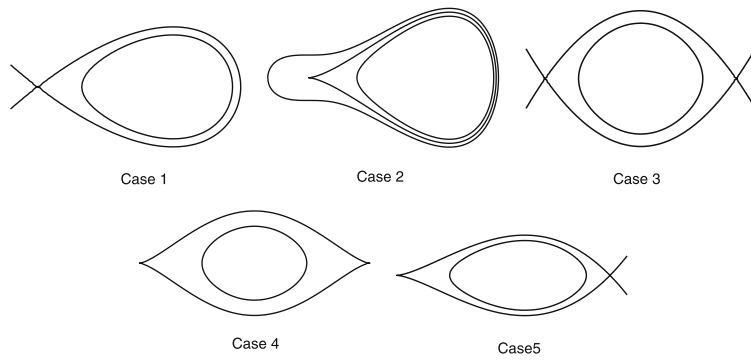


Fig. 1. The phase portraits of 5 cases of the boundary.

- (4) It is a heteroclinic loop having 2 heteroclinic orbits connecting 2 cusps.
- (5) It is a heteroclinic loop having 2 heteroclinic orbits connecting 1 cusp and 1 hyperbolic saddle. See Fig. 1.

The expansions of function M were obtained for the first three cases in [4–6]. In this paper we discuss the last two cases, obtaining the expansion of M .

Before we state our main results we first list relevant results for the first 3 cases.

For case (1) we can suppose the homoclinic loop L_0 is given by $\{(x, y) | H(x, y) = 0\}$ and the hyperbolic saddle is at the origin. Then without loss of generality we may suppose

$$H(x, y) = \frac{\lambda}{2}(y^2 - x^2) + \sum_{i+j \geq 3} h_{ij}x^i y^j, \quad \lambda \neq 0 \tag{1.4}$$

for (x, y) near the origin. For definiteness, we assume that L_h exists for $0 < -h \ll 1$.

Theorem 1.1. *Under the above assumptions we have*

$$M(h, \delta) = \sum_{j \geq 0} (c_{2j} + c_{2j+1}h \ln |h|)h^j$$

for $0 < -h \ll 1$ [4]. Further, for the formula of c_0, c_1, c_2 and c_3 , we have [5]

$$\begin{aligned} c_0(\delta) &= M(0, \delta) = \oint_{L_0} qdx - pdy, \\ c_1(\delta) &= -\frac{a_{10} + b_{01}}{|\lambda|}, \\ c_2(\delta) &= \oint_{L_0} (p_x + q_y - a_{10} - b_{01})|_{\varepsilon=0} dt + bc_1(\delta), \\ c_3(\delta) &= \frac{-1}{\lambda|\lambda|}(a_{21} + b_{12}) - \frac{1}{\lambda}[h_{12}(2a_{20} + b_{11}) + h_{21}(a_{11} + 2b_{02})] + \bar{b}c_1(\delta), \end{aligned}$$

where b and \bar{b} are constants.

As in [5], $c_1(\delta)$ and $c_3(\delta)$ are called the local homoclinic coefficients of M at the saddle O , denoted by $c_1(O_{\text{saddle}}, \delta)$ and $c_3(O_{\text{saddle}}, \delta)$ respectively. By Han et al. [5], we have directly

Corollary 1.1 ([5]). *Let system (1.1) satisfy (1.4). Let U denote a disk of diameter $\varepsilon_0 > 0$ with center at the saddle. Then*

$$\begin{aligned} \lim_{h \rightarrow 0} \int_{L_h \cap U} (p_x + q_y - \sigma)|_{\varepsilon=0} dt &= \int_{L_0 \cap U} (p_x + q_y - \sigma)|_{\varepsilon=0} dt, \\ \lim_{h \rightarrow 0} \left[\int_{L_h \cap U} dt + \frac{1}{|\lambda|} \ln |h| \right] &= O(1) \in \mathbb{R}, \end{aligned}$$

where $\sigma = (p_x + q_y)|_{x=y=0}$, which is called the divergence of the saddle.

For case (2), we also suppose the homoclinic loop $L_0 \subset \{(x, y) | H(x, y) = 0\}$ and the cusp is at the origin. Then for the simplest case of cusp, we suppose

$$H(x, y) = \frac{1}{2}y^2 + \sum_{i+j \geq 3} h_{ij}x^i y^j, \quad h_{30} \neq 0. \tag{1.5}$$

Download English Version:

<https://daneshyari.com/en/article/841184>

Download Persian Version:

<https://daneshyari.com/article/841184>

[Daneshyari.com](https://daneshyari.com)