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Well-posedness, stability and invariance results for a class of multivalued Lur'e dynamical systems

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ABSTRACT

This paper analyzes the existence and uniqueness issues in a class of multivalued Lur'e systems, where the multivalued part is represented as the subdifferential of some convex, proper, lower semicontinuous function. Through suitable transformations the system is recast into the framework of dynamic variational inequalities and the well-posedness (existence and uniqueness of solutions) is proved. Stability and invariance results are also studied, together with the property of continuous dependence on the initial conditions. The problem is motivated by practical applications in electrical circuits containing electronic devices with nonsmooth multivalued voltage/current characteristics, and by state observer design for multivalued systems.

1. Introduction

Lur'e systems, which consist of a linear time-invariant system in negative feedback with a static nonlinearity satisfying a sector condition, have received a considerable interest in the applied mathematics and control literature, due to their broad interest (see [1] for a survey). More recently the case where the nonlinearity is a maximal monotone map has been studied [2]. The maximal monotonicity allows one to consider unbounded sectors $[0, +\infty]$ and nonsmooth set-valued nonlinearities. So-called linear complementarity systems can be recast into Lur'e systems, where the feedback nonlinearity takes the form of a set of complementarity conditions between two slack variables [3–5]. One of these slack variables may be interpreted as a Lagrange multiplier λ, while the other one usually takes the form $y = Cx + D\lambda$. More general piecewise linear nonlinearities have been considered in [6,7]. As pointed out in [2] there exists a close relationship between some complementarity systems and differential inclusions with maximal monotone right-hand sides, in particular inclusions into normal cones to convex sets (which are in turn equivalent to dynamical variational inequalities of the first kind). Particular cases have been investigated in [8–10]. All these works are however restricted to the case where $D = 0$, except [7] where affine complementarity systems are considered. In this paper, we extend the works in [8,9] to the case where $D \neq 0$, i.e. there exists a feedthrough matrix in the linear part of the system. Moreover the nonlinearities which we consider are much more general than complementarity conditions between y and λ (i.e. $y \geq 0$, $\lambda \geq 0$, $y^T\lambda = 0$) and the considered systems may be written equivalently as dynamical variational inequalities of the second kind. Such an extension may be important in practice (for instance electrical circuits with ideal diodes and transistors usually yield systems with a nonzero feedthrough matrix D, possibly positive semi-definite and non-symmetric). Observer synthesis for set-valued systems is also an important application [11,12]. This work may also be seen as the continuation of previous efforts to study the relationships

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between various types of differential inclusions, complementarity systems, projected systems in finite dimensions [10,13–15].

The paper is organized as follows: In Section 2 the dynamical system is presented, and its well-posedness is studied in Section 3. In Section 4 the stability properties are studied, and an invariance result is presented in Section 5. Conclusions end the paper in Section 6.

Notations: Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a proper convex and lower semicontinuous function, we denote by $\mathrm{dom}(f) := \{x \in \mathbb{R}^n : f(x) < +\infty\}$ the domain of the function $f(\cdot)$. Recall that the Fenchel transform $f^*(\cdot)$ of $f(\cdot)$ is the proper, convex and lower semicontinuous function defined by

$$(\forall z \in \mathbb{R}^n) : f^*(z) = \sup_{x \in \mathrm{dom}(f)} \{\langle x, z \rangle - f(x)\}.$$

The subdifferential $\partial f(x)$ of $f(\cdot)$ at $x \in \mathbb{R}^n$ is defined by

$$\partial f(x) = \{\omega \in \mathbb{R}^n : f(v) - f(x) \geq \langle \omega, v - x \rangle, \forall v \in \mathbb{R}^n\},$$

where $\langle \cdot, \cdot \rangle$ denotes the usual scalar product in $\mathbb{R}^n$, i.e. $\langle y, z \rangle = y^T z$ for any vectors y and z of $\mathbb{R}^n$. We denote by $\mathrm{Dom}(\partial f) := \{x \in \mathbb{R}^n : \partial f(x) \neq \emptyset\}$ the domain of the subdifferential operator $\partial f : \mathbb{R}^n \to \mathbb{R}^n$. Let x_0 be any element in the domain $\mathrm{dom}(f)$ of $f(\cdot)$, the recession function $f_\infty(\cdot)$ of $f(\cdot)$ is defined by

$$(\forall x \in \mathbb{R}^n) : f_\infty(x) = \lim_{\lambda \to +\infty} \frac{1}{\lambda} f(x_0 + \lambda x).$$

The function $f_\infty : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ is a proper convex and lower semicontinuous function which describes the asymptotic behavior of $f(\cdot)$. For a nonempty closed and convex set $K \subset \mathbb{R}^n$, the dual cone of K is the nonempty closed convex cone $K^\star$ defined by

$$K^\star := \{w \in \mathbb{R}^n : \langle w, v \rangle \geq 0, \forall v \in K\}, \tag{1}$$

while the polar cone $K^o = -K^\star$. Let x_0 be any element in K, the recession cone of K is defined by

$$K_\infty = \bigcap_{\lambda > 0} \frac{1}{\lambda}(K - x_0).$$

The set K_∞ is a nonempty closed convex cone that is described in terms of the directions which recede from K. When K is a cone then $K_\infty = K$. The relative interior of a set K is denoted as rint (K), and its closure as $\bar{K}$. Let $M \in \mathbb{R}^{m \times n}$ be a given matrix, we denote by $\ker(M)$ the kernel of M and by $\mathcal{R}(M)$ the range of M. $M \geq 0$ means that M is positive semidefinite, $M > 0$ means that it is positive definite.

2. The multivalued Lur'e system

Let $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times p}$, $C \in \mathbb{R}^{p \times n}$, $D \in \mathbb{R}^{p \times p}$ be given matrices, $f \in C^0(\mathbb{R}_+; \mathbb{R})$ such that $f' \in L^1_{\mathrm{loc}}(\mathbb{R}_+; \mathbb{R}^n)$ and $\varphi_i : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$ $(1 \leq i \leq p)$ given proper convex and lower semicontinuous functions. Let $x_0 \in \mathbb{R}^n$ be some initial condition, we consider the problem: Find $x \in C^0(\mathbb{R}_+; \mathbb{R}^n)$ such that $x' \in L^\infty_{\mathrm{loc}}(\mathbb{R}_+; \mathbb{R}^n)$ and x right-differentiable on $\mathbb{R}_+$, $\lambda \in C^0(\mathbb{R}_+; \mathbb{R}^p)$ and $y \in C^0(\mathbb{R}_+; \mathbb{R}^p)$ satisfying the nonsmooth dynamical system $NSDS(A, B, C, D, f, \varphi_1, \ldots, \varphi_p, x_0)$:

$$\begin{cases} x(0) = x_0 \quad \text{a.e. } t \geq 0 \\ x'(t) = Ax(t) + B\lambda(t) + f(t) \quad \text{for all } t \geq 0 \\ y(t) = Cx(t) + D\lambda(t) \\ \lambda_1(t) \in -\partial\varphi_1(y_1(t)) \\ \lambda_2(t) \in -\partial\varphi_2(y_2(t)) \\ \vdots \\ \lambda_p(t) \in -\partial\varphi_p(y_p(t)). \end{cases} \tag{2}$$

The system is therefore in the canonical absolute stability form since it is the negative feedback interconnection of a linear invariant system (A, B, C, D) (with "input" λ, "output" y and external excitation $f(\cdot)$) with a static multivalued nonlinearity (with "input" y and "output" $-\lambda$). In [8,9] it was considered that $D = 0$. As we shall see next the case $D \neq 0$ complicates the analysis. It is noteworthy that one may have $p > n$, which is crucial because λ is not a control input and p may in applications be very large. Physical examples are given later in the paper. It is assumed in this paper that the "output" y does not depend explicitly on time. If this is the case the results of this paper do not apply because one has to resort to the perturbed Moreau's sweeping process to derive well-posedness results, see [16].

Let us set $\lambda = (\lambda_1 \ \cdots \ \lambda_p)^T$, $\Phi(\cdot) = \varphi_1(\cdot) + \cdots + \varphi_p(\cdot)$, and $M \in \mathbb{R}^{p \times p}$ is an invertible matrix. One may consider a slightly more general version of the Lur'e system (2) as:

$$\begin{cases} x(0) = x_0 \\ x'(t) = Ax(t) + B\lambda(t) + f(t) \\ \lambda(t) \in -M\partial\Phi(y(t)). \end{cases} \tag{3}$$