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Asymptotic behavior for nonlocal dispersal equations☆

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ABSTRACT

This paper is concerned with the existence and asymptotic behavior of solutions of a nonlocal dispersal equation. By means of super-subsolution method and monotone iteration, we first study the existence and asymptotic behavior of solutions for a general nonlocal dispersal equation. Then, we apply these results to our equation and show that the nonnegative solution is unique, and the behavior of this solution depends on parameter λ in equation. For $\lambda \leq \lambda_1(\Omega)$, the solution decays to zero as $t \to \infty$; while for $\lambda > \lambda_1(\Omega)$, the solution converges to the unique positive stationary solution as $t \to \infty$. In addition, we show that the solution blows up under some conditions.

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1. Introduction

In this paper, we consider the following nonlocal dispersal problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = J * u - u + \lambda u - a(x)f(u), & (x,t) \in \Omega \times (0,+\infty), \\ u(x,t) = 0, & (x,t) \in (\mathbb{R}^N \setminus \Omega) \times [0,+\infty), \\ u(x,0) = \phi(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ and λ is a real parameter. The operator $J * u - u$ is a nonlocal dispersal operator, $J * u$ is the usual convolution defined by

$$(J * u)(x,t) = \int_{\mathbb{R}^N} J(x-y)u(y,t)\mathrm{d}y.$$

The kernel $J : \mathbb{R}^N \to \mathbb{R}$ satisfies: $J \in C^1(\mathbb{R}^N), J \geq 0, J(x) = J(-x), \int_{\mathbb{R}^N} J(x)\mathrm{d}x = 1$, and has a compact support. The function $a(x)$ may be positive in Ω or zero in a certain subdomain Ω_0 of Ω, $f \in C^1(\mathbb{R})$ and the initial function $\phi(x) \in C(\overline{\Omega})$ is nonnegative.

The problem (1.1) was proposed to describe the spatial dispersal of a single species in Ω [1–3]. $u(x,t)$ represents the density of the species at location x at time t. If $J(x-y)$ is thought of as the probability distribution of jumping from

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location y to location x, then the rate at which the individuals arriving to location x from all other places is $(J * u)(x,t) = \int_{\mathbb{R}^N} J(x-y)u(y,t)\mathrm{d}y$. On the other hand, the rate at which they are leaving location x to travel to all other places is $-u(x,t) = -\int_{\mathbb{R}^N} J(x-y)u(x,t)\mathrm{d}y$. λ is the birth rate and $a(x)$ measures the intraspecific competition or saturation. If $a(x)=0$ in Ω_0, we can consider Ω_0 as a "refuge place" for the species, since it is free from competition there.

In this paper, we are interested in the existence and asymptotic behavior of the solutions of (1.1). As we know, in the study of nonlocal dispersal equations, there are many results on traveling wave solutions [4–6,2,7–10], but little on the asymptotic behavior of the global solutions. Chasseigne et al. [1] and Cortazar et al. [11] studied, respectively, the existence and asymptotic behavior of the solutions of a nonlocal dispersal equation without nonlinear terms in the whole $\mathbb{R}^N$ or in a bounded smooth domain with Dirichlet or Neumann boundary condition. Later, Pazoto and Rossi [12] paid attention to the same problem for a nonlocal dispersal equation in the whole $\mathbb{R}^N$ with an absorptive nonlinear term, and showed the influence of the absorption term on the behavior of solutions. Other results on nonlocal dispersal equations with Neumann boundary condition or Dirichlet boundary condition can be seen in [13–16] and references therein.

In order to investigate the problem (1.1), it is necessary to recall its local analogue

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + \lambda u - a(x)f(u), & (x,t) \in \Omega \times (0,+\infty), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times [0,+\infty), \\ u(x,t) = \phi(x), & x \in \Omega. \end{cases} \tag{1.2}$$

In 1972, Sattinger [17] investigated the existence and asymptotic behavior of the solution for a more general class of (1.2) without initial condition, and gave a number of examples as application. One of those examples is the stationary state problem of (1.2) with $a(x) = 1$ and $f(u) = u^3$. In the paper, he showed that the stability of stationary solutions of (1.2) depends on parameter λ. When $\lambda < \lambda_1(\Omega)$, the null solution is stable; when $\lambda > \lambda_1(\Omega)$, there exist stable positive and negative (nontrivial) stationary solutions, where $\lambda_1(\Omega)$ is the first Dirichlet eigenvalue of Laplacian in Ω.

Since nonlocal dispersal operator shares many properties of the classical Laplacian operator (see [18,1,11,19]), such as, both of them have maximum principle, bounded stationary solutions are constant, and even if J is compactly supported, perturbations propagate with infinite speed, a natural question arises: do the results valid for (1.2) persist when dealing with (1.1)?

The answer is affirmative. We will show that the nonlocal problem (1.1) has a similar asymptotic behavior as local problem (1.2) by taking the method of super-subsolution. In addition, we get an important result different from the local case, that is, the integral of the solution of (1.1) in $\Omega_0 \subset \Omega$ will blow up for $\lambda > \lambda_1(\Omega_0)$ and $a(x) = 0$ in Ω_0.

The rest of this paper is organized as follows. In Section 2, we first state some results for the nonlocal operator: the maximum principle, the comparison principle and the eigenvalue problem. Then, we establish the existence of positive solution to the steady-state problem of (1.1). In Section 3, we show the existence and asymptotic behavior of solutions to a general nonlocal dispersal problem. The existence and asymptotic behavior of solution to (1.1) will be provided in Section 4.

2. Preliminaries

In this section, we first establish some technical lemmas which will be used in the remainder of this paper.

Lemma 2.1 (*Maximum Principle*). *Suppose that* $u \in C^1([0,+\infty); C(\overline{\Omega}))$ *and* $u(x,t)$ *satisfies*

$$\begin{cases} u_t(x,t) \geq (J * u - u)(x,t) + K(x,t)u(x,t), & (x,t) \in \Omega \times (0,+\infty), \\ u(x,t) \geq 0, & (x,t) \in (\mathbb{R}^N \setminus \Omega) \times [0,+\infty), \\ u(x,0) \geq 0, & x \in \Omega, \end{cases} \tag{2.1}$$

where $K(x,t) : \Omega \times [0,+\infty) \to \mathbb{R}$ *is continuous and uniformly bounded. Then,* $u(x,t) \geq 0$ *for* $x \in \mathbb{R}^N$ *and* $t \in [0,+\infty)$. *Moreover, if* $u(x,0) \not\equiv 0$ *for* $x \in \Omega$, *then* $u(x,t) > 0$ *for* $x \in \Omega$ *and* $t \in (0,+\infty)$.

Proof. Set $v(x,t) = \mathrm{e}^{\mu t}u(x,t)$. Then it satisfies

$$\begin{cases} v_t(x,t) \geq (J * v)(x,t) + [\mu + K(x,t) - 1]v(x,t), & (x,t) \in \Omega \times (0,+\infty), \\ v(x,t) \geq 0, & (x,t) \in (\mathbb{R}^N \setminus \Omega) \times [0,+\infty), \\ v(x,0) \geq 0, & x \in \Omega, \end{cases} \tag{2.2}$$

where $\mu > 0$ such that $\mu + K(x,t) - 1 > 0$.

Let

$$K_0 = \sup_{x\in\Omega, t\in[0,+\infty)} K(x,t).$$

Take $\tau = \frac{1}{2}(\mu + K_0)^{-1}$. Suppose that for some $x \in \Omega$ and $t \in [0,\tau]$, $u(x,t) < 0$. Then, obviously, $v(x,t) < 0$ at the same point. Hence, there exist $x_0 \in \Omega$ and $t_0 \in [0,\tau]$ such that

$$\min_{x\in\Omega, t\in[0,\tau]} v(x,t) = v(x_0,t_0) < 0.$$

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