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Fixed point theorems in fuzzy metric spaces using property E.A.

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ABSTRACT

We prove two common fixed point theorems for a pair of weakly compatible maps in fuzzy metric spaces both in the sense of Kramosil and Michalek and in the sense of George and Veeramani, by using E.A. property.

1. Introduction and preliminaries

The notion of fuzzy set, which laid the foundation of fuzzy mathematics was introduced by Zadeh in [1]. There are many viewpoints of the notion of metric space; see, e.g., [2–4]. For the reader's convenience we recall some terminologies from the theory of fuzzy metric spaces, which will be used in what follows.

Definition 1.1. A *continuous t-norm* (in the sense of Schweizer and Sklar [5]) is a binary operation $*$ on $[0, 1]$ satisfying the following conditions:

(i) $*$ is commutative and associative;
(ii) $a * 1 = a$ for all $a \in [0, 1]$;
(iii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ ($a, b, c, d \in [0, 1]$);
(iv) the mapping $* : [0, 1] \times [0, 1] \to [0, 1]$ is continuous.

As classical examples of continuous t-norms we mention the t-norms T_L, T_P, T_M, defined through $T_L(a, b) = max(a + b - 1, 0)$, $T_P(a, b) = ab$ and $T_M(a, b) = min(a, b)$.

Definition 1.2. A *fuzzy metric space in the sense of Kramosil and Michalek* [3] is a triple $(X, M, *)$ where X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ such that the following axioms hold:

(FM-1) $M(x, y, 0) = 0$ $(x, y \in X)$.
(FM-2) $M(x, y, t) = 1$ $\forall t > 0$ iff $x = y$;
(FM-3) $M(x, y, t) = M(y, x, t)$ $(x, y \in X, t > 0)$;

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(FM-4) $M(x, y, \cdot)$: $[0, \infty) \to [0, 1]$ is left continuous $\forall x, y \in X$;
(FM-5) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$ $\forall x, y, z \in X, \forall t, s > 0$.

We will refer to these spaces as *KM-fuzzy metric spaces*.

Lemma 1.3 (*[6]*). *For every $x, y \in X$, the mapping $M(x, y, \cdot)$ is nondecreasing on $(0, \infty)$.*

Definition 1.4. A *fuzzy metric space in the sense of George and Veeramani* [4] is a triple $(X, M, *)$ where X is a nonempty set, $*$ is a continuous t-norm, M is a fuzzy set on $X^2 \times (0, \infty)$ and the following conditions are satisfied for all $x, y \in X$ and $t, s > 0$:

(GV-1) $M(x, y, t) > 0$;
(GV-2) $M(x, y, t) = 1$ iff $x = y$;
(GV-3) $M(x, y, t) = M(y, x, t)$;
(GV-4) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous;
(GV-5) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$.

It is worth noticing that, from (GV-2) and (GV-1) it follows that if $x \neq y$, then $0 < M(x, y, t) < 1$ for all $t > 0$ [7]. In what follows, fuzzy metric spaces in the sense of George and Veeramani will be called *GV-fuzzy metric spaces*.

Example 1.5 (*[4]*). Let (X, d) be a metric space, $a * b = T_M(a, b)$ and, for all x, y in X and $t > 0$,

$$M(x, y, t) = \frac{t}{t + d(x, y)}.$$

Then $(X, M, *)$ is a GV-fuzzy metric space, called *standard fuzzy metric space* induced by (X, d).

Definition 1.6. Let $(X, M, *)$ be a (KM- or GV-) fuzzy metric space. A sequence $(x_n)_{n\in\mathbb{N}}$ in X is said to be *convergent* to $x \in X$ if

$$\lim_{n\to\infty} M(x_n, x, t) = 1$$

for all $t > 0$.

A sequence $\{x_n\}$ in X is said to be a *G-Cauchy sequence* [6,8] if

$$\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$$

for all $t > 0$ and $p \in \mathbb{N}$.

A fuzzy metric space is called G-complete if every G-Cauchy sequence converges in X.

Lemma 1.7 (*cf. [5]*). *If $(X, M, *)$ is a KM-fuzzy metric space and $\{x_n\}$, $\{y_n\}$ are sequences in X such that $x_n \to x$, $y_n \to y$, then $M(x_n, y_n, t) \to M(x, y, t)$ for every continuity point t of $M(x, y, \cdot)$.*

Fixed point theory in fuzzy metric spaces has been developed starting with the paper of Grabiec [6]. Subrahmanyam [9] gave a generalization of Jungk's [10] common fixed point theorem for commuting mappings in the setting of fuzzy metric spaces. Even if in the recent literature weaker conditions of commutativity, as weakly commuting mappings, compatible mappings, R-weakly commuting maps, weakly compatible mappings and several others have been utilizing, the existence of a common fixed point requires some conditions on continuity of the maps, G-completeness of the space, or containment of ranges.

The concept of E.A. property in a metric space has been recently introduced by Aamri and Moutawakil.

Definition 1.8 (*[11]*). Two self-maps A and S of a metric space (X, d) are said to satisfy *E.A. property* if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$$

for some $t \in X$.

In a similar mode, it is said that two self-maps and A and S of a fuzzy metric space $(X, M, *)$ satisfy E.A. property, if there exist a sequence $\{x_n\}$ in X and z in X such that Ax_n and Sx_n converge to z in the sense of Definition 1.6.

Some common fixed point theorems in probabilistic or fuzzy metric spaces by E.A. property under weak compatibility have been recently obtained in [12–14]. It was pointed out in [15], that this property buys containment of ranges without any continuity requirements, besides minimize the commutativity conditions of the maps to the commutativity at their points of coincidence. Moreover, E.A. property allows to replace the completeness requirement of the space with a more natural condition of closeness of the range (it turns out that G-completeness is a quite strong kind of completeness, see, e.g., [16,17]).