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Global existence of solutions for the one-dimensional motions of a compressible viscous gas with radiation: An "infrarelativistic model"

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ABSTRACT

We consider an initial–boundary value problem for the equations of 1D motions of a compressible viscous heat-conducting gas coupled with radiation through a radiative transfer equation. Assuming suitable hypotheses on the transport coefficients, we prove that the problem admits a unique weak solution.

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1. Introduction

The aim of radiation hydrodynamics is to include the effects of radiation into the hydrodynamical framework. When equilibrium holds between the matter and the radiation, a simple way to do that is to include local radiative terms into the state functions and the transport coefficients. One knows from quantum mechanics that radiation is described by its quanta, the photons, which are massless particles traveling at the speed c of light, characterized by their frequency ν, their energy $E = h\nu$ (where h is Planck's constant), and their momentum $\vec{p} = \frac{\hbar\nu}{c}\vec{\Omega}$, where $\vec{\Omega}$ is a unit vector. Statistical mechanics allows us to describe macroscopically an assembly of massless photons of energy E and momentum $\vec{p}$ by using a distribution function: the radiative intensity $I(r, t, \vec{\Omega}, \nu)$. Using this fundamental quantity, one can derive global quantities by integrating with respect to the angular and frequency variables: the spectral radiative energy density $E_R(r, t)$ per unit volume is then $E_R(r, t) := \frac{1}{c}\iint I(r, t, \vec{\Omega}, \nu)\,d\Omega\,d\nu$, and the spectral radiative flux $\vec{F}_R(r, t) = \iint \vec{\Omega}\, I(r, t, \vec{\Omega}, \nu)\,d\Omega\,d\nu$. If matter is in thermodynamic equilibrium at constant temperature T and if radiation is also in thermodynamic equilibrium with matter, its temperature is also T and statistical mechanics tells us that the distribution function for photons is given by the Bose–Einstein statistics with zero chemical potential.

In the absence of radiation, one knows that the complete hydrodynamical system is derived from the standard conservation laws of mass, momentum and energy by using Boltzmann's equation satisfied by the $f_m(r, \vec{v}, t)$ and the

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Chapman–Enskog expansion [1]. One gets then formally the compressible Navier–Stokes system

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{u}) = 0, \\ \partial_t (\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) = -\nabla \cdot \overset{\Rightarrow}{\Pi} + \vec{f}, \\ \partial_t (\rho \varepsilon) + \nabla \cdot (\rho \varepsilon \vec{u}) = -\nabla \vec{q} - \overset{\Rightarrow}{D} : \overset{\Rightarrow}{\Pi} + g, \end{cases} \tag{1}$$

where $\overset{\Rightarrow}{\Pi} = -p(\rho, T) \ \overset{\Rightarrow}{I} + \overset{\Rightarrow}{\pi}$ is the material stress tensor for a newtonian fluid with the viscous contribution $\overset{\Rightarrow}{\pi} = 2\mu \ \overset{\Rightarrow}{D} + \lambda \nabla \cdot \vec{u} \ \overset{\Rightarrow}{I}$ with $3\lambda + 2\mu \geqslant 0$ and $\mu > 0$, and the strain tensor $\overset{\Rightarrow}{D}$ such that $\overset{\Rightarrow}{D}_{ij}(\vec{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$. $\vec{q}$ is the thermal heat flux and $\vec{F}$ and g are external force and energy source terms.

When radiation is present, the terms $\vec{f}$ and g include the terms for the coupling between the matter and the radiation (neglecting any other external field), depending on I, and I is driven by a transport equation: the so called radiative transfer integro-differential equation discussed by Chandrasekhar in [2].

Supposing that the matter is at LTE, the coupled system reads

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{u}) = 0, \\ \partial_t (\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) = -\nabla \cdot \overset{\Rightarrow}{\Pi} - \vec{S}_F, \\ \partial_t (\rho \varepsilon) + \nabla \cdot (\rho \varepsilon \vec{u}) = -\nabla \vec{q} - \overset{\Rightarrow}{D} : \overset{\Rightarrow}{\Pi} - S_E, \\ \dfrac{1}{c} \dfrac{\partial}{\partial t} I\left(r, t, \vec{\Omega}, \nu\right) + \vec{\Omega} \cdot \nabla I\left(r, t, \vec{\Omega}, \nu\right) = S_t\left(r, t, \vec{\Omega}, \nu\right), \end{cases} \tag{2}$$

where the coupling terms are

$$S_t(r, t, \vec{\Omega}, \nu) = \sigma_a\left(\nu, \vec{\Omega}, \rho, T, \frac{\vec{\Omega} \cdot \vec{u}}{c}\right)\left[B(\nu, T) - I\left(r, t, \vec{\Omega}, \nu\right)\right] + \iint \sigma_s\left(r, t, \rho, \vec{\Omega}' \cdot \vec{\Omega}, \nu' \to \nu\right)$$
$$\times \left\{\frac{\nu}{\nu'} I\left(r, t, \vec{\Omega}', \nu'\right) I\left(r, t, \vec{\Omega}, \nu\right) - \sigma_s\left(r, t, \rho, \vec{\Omega} \cdot \vec{\Omega}', \nu \to \nu'\right) I\left(r, t, \vec{\Omega}, \nu\right) I\left(r, t, \vec{\Omega}', \nu'\right)\right\} \mathrm{d}\Omega' \, \mathrm{d}\nu',$$

the radiative energy source

$$S_E(r, t) := \iint S_t(r, t, \vec{\Omega}, \nu) \, \mathrm{d}\Omega \, \mathrm{d}\nu,$$

the radiative flux

$$\vec{S}_F(r, t) := \frac{1}{c} \iint \vec{\Omega} \, S_t(r, t, \vec{\Omega}, \nu) \, \mathrm{d}\Omega \, \mathrm{d}\nu.$$

In the radiative transfer equation (the last Eq. (2)) the functions σ_a and σ_s describe in a phenomenological way the absorption–emission and scattering properties of the photon–matter interaction, and Planck's function $B(\nu, \theta)$ describes the frequency–temperature black body distribution.

Let us note that the foundations for the previous system were described by Pomraning [3] and Mihalas and Weibel-Mihalas [4] in the full framework of special relativity (oversimplified in the previous considerations). The coupled system (2) has been investigated recently (in the inviscid case) by Lowrie, Morel and Hittinger [5], Buet and Després [6] with special attention paid to asymptotic regimes, and by Dubroca and Feugeas [7], Lin [8] and Lin, Coulombel and Goudon [9] as regards numerical aspects. As regards the existence of solutions, a proof of local-in-time existence and blow-up of solutions (in the inviscid case) has been proposed by Zhong and Jiang [10] (see also the recent papers by Jiang and Wang [11,12] for a 1D related "Euler–Boltzmann" model). Moreover, a simplified version of the system has been investigated by Golse and Perthame [13].

As the multidimensional viscous situation is far from been understood even at the formal level (however see [14] for a macroscopic treatment of radiation in the astrophysical context, and [15] for the associated mathematical treatment), we restrict the following to the monodimensional case.

In 1D the previous system reads

$$\begin{cases} \rho_\tau + (\rho v)_y = 0, \\ (\rho v)_\tau + (\rho v^2)_y + p_y = \mu v_{yy} - (S_F)_R, \\ \left[\rho\left(e + \dfrac{1}{2} v^2\right)\right]_\tau + \left[\rho v\left(e + \dfrac{1}{2} v^2\right) + pv - \kappa \theta_y - \mu v v_y\right]_y = -(S_E)_R, \\ \dfrac{1}{c} I_t + \omega I_y = S. \end{cases} \tag{3}$$

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