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Nondifferentiable multiobjective symmetric duality with F-convexity over cones

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ABSTRACT

In the present paper, a pair of Wolfe type nondifferentiable multiobjective second-order symmetric dual programs over arbitrary cones are formulated. Using the concept of weak efficiency with respect to a convex cone, weak, strong and converse duality theorems are studied under second-order K–F-convexity assumptions. Self-duality is also discussed.

1. Introduction

The duality in linear programming is symmetric, i.e., the dual of the dual is the primal problem. This is not the case in nonlinear programming in general. Dorn [1] introduced the concept of symmetric duality in quadratic programming. His results were extended to general nonlinear programs involving convex/concave functions by Dantzig et al. [2] and then by Bazaraa and Goode [3] over cone constraints. Chandra et al. [4] studied symmetric duality in mathematical programming under F-convexity/F-pseudoconvexity for Wolfe and Mond–Weir type models. Kim et al. [5] constructed a pair of multiobjective symmetric dual programs for pseudo-invex functions over arbitrary cones and obtained various duality results. Multiobjective symmetric dual programs over cones in which the objective function is optimized with respect to a cone have been discussed in [6–9]. Recently, Kim and Lee [10] studied nondifferentiable higher-order multiobjective dual programs involving cone constraints and established duality results under higher-order generalized convexity assumptions.

Mangasarian [11] introduced the concept of second-order duality in nonlinear programming. He indicated that it provides tighter bounds for the value of objective functions. This motivated several researchers in this field. Second-order symmetric duality involving nondifferentiable functions has been discussed by Hou and Yang [12] for Mond–Weir type duals, and by Ahmad and Husain [13] and Yang et al. [14] for Wolfe type duals. Yang et al. [15], and Gupta and Kailey [16] studied multiobjective second-order symmetric duality under F-convexity.

In this paper, we have formulated Wolfe type nondifferentiable second-order multiobjective symmetric dual programs over arbitrary cones. Using the concept of weak efficiency with respect to a convex cone, weak, strong and converse duality theorems have been established under second-order K–F-convexity assumptions. Some of the known results are obtained as special cases. Self-duality for our programs has also been discussed.

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2. Notation and preliminaries

We consider the following multiobjective programming problem:

$$K\text{-minimize } \phi(x) \tag{P}$$
$$\text{subject to } -g(x) \in Q, \quad x \in S,$$

where $S \subseteq R^n$, $\phi : R^n \to R^k$, $g : R^n \to R^m$, and K and Q are closed convex pointed cones with non-empty interior in R^k and R^m, respectively. Let $X = \{x \in S : -g(x) \in Q\}$ be the set of all feasible solutions of (P). Further, let K_0 denote the set $K \setminus \{0\}$. All the vectors will be considered as column vectors.

Definition 2.1. A point $\bar{x} \in X$ is said to be an efficient (a weakly efficient) solution of (P) if there exists no $x \in X$ such that $\phi(\bar{x}) - \phi(x) \in K_0$ (int K).

Let C_1 and C_2 be closed convex cones in R^n and R^m, respectively. Also, let $S_1 \subseteq R^n$ and $S_2 \subseteq R^m$ be open sets such that $C_1 \times C_2 \subset S_1 \times S_2$.

Definition 2.2. The positive polar cone C_i^* of C_i $(i = 1, 2)$ is defined as $C_i^\star = \{z : x^T z \geqq 0,\ \text{for all } x \in C_i\}$.

Definition 2.3. A functional $F : S_1 \times S_1 \times R^n \longrightarrow R$ is said to be sublinear in the third variable if for all $x, u \in S_1$,

(i) $F(x, u; a_1 + a_2) \leqq F(x, u; a_1) + F(x, u; a_2)$ for all $a_1, a_2 \in R^n$, and
(ii) $F(x, u; \alpha a) = \alpha F(x, u; a)$ for all $\alpha \in R_+$ and $a \in R^n$.

For notational convenience, we will write $F_{(x,u)}(a)$ for $F(x, u; a)$.

Definition 2.4 (*[17]*)**.** A twice-differentiable function $f : S_1 \times S_2 \mapsto R^k$ is said to be second-order K–F-convex in the first variable at $u \in S_1$ for fixed $v \in S_2$ if there exists a sublinear functional $F : S_1 \times S_1 \times R^n \longrightarrow R$ such that for $x \in S_1,\ q_i \in R^n,\ i = \{1, 2, \ldots, k\}$,

$$\Big(f_1(x, v) - f_1(u, v) + \frac{1}{2}{q_1}^T \nabla_{xx} f_1(u, v) q_1 - F_{(x,u)}[\nabla_x f_1(u, v) + \nabla_{xx} f_1(u, v) q_1], \ldots, f_k(x, v) - f_k(u, v)$$
$$+ \frac{1}{2}{q_k}^T \nabla_{xx} f_k(u, v) q_k - F_{(x,u)}[\nabla_x f_k(u, v) + \nabla_{xx} f_k(u, v) q_k]\Big) \in K,$$

and $f(x, y)$ is said to be second-order K–G-concave in the second variable at $v \in S_2$ for fixed $u \in S_1$ if there exists a sublinear functional $G : S_2 \times S_2 \times R^m \longrightarrow R$ such that for $y \in S_2,\ p_i \in R^m,\ i = \{1, 2, \ldots, k\}$,

$$\Big(-f_1(u, y) + f_1(u, v) - \frac{1}{2} p_1^T \nabla_{yy} f_1(u, v) p_1 + G_{(y,v)}[\nabla_y f_1(u, v) + \nabla_{yy} f_1(u, v) p_1], \ldots, -f_k(u, y) + f_k(u, v)$$
$$- \frac{1}{2} p_k^T \nabla_{yy} f_k(u, v) p_k + G_{(y,v)}[\nabla_y f_k(u, v) + \nabla_{yy} f_k(u, v) p_k]\Big) \in K.$$

Lemma 2.1 (*Generalized Schwarz Inequality*)**.** *Let $A \in R^n \times R^n$ be a positive semi-definite matrix. Then for all $x, y \in R^n$,*

$$x^T A y \leqq (x^T A x)^{\frac{1}{2}} (y^T A y)^{\frac{1}{2}}.$$

Equality holds if $Ax = \lambda A y$ for some $\lambda \geqq 0$.

Now we consider the following pair of Wolfe type nondifferentiable multiobjective programming problems. It may be noted that since K is a pointed cone, int K^* is non-empty.

Primal problem (WP):

$$K\text{-minimize } H(x, y, \lambda, p) = f(x, y) + (x^T D x)^{\frac{1}{2}} e_k - y^T \nabla_y (\lambda^T f)(x, y) e_k$$
$$- y^T (\nabla_{yy} (\lambda^T f)(x, y) p) e_k - \frac{1}{2} p^T (\nabla_{yy} (\lambda^T f)(x, y) p) e_k,$$

subject to

$$-\nabla_y (\lambda^T f)(x, y) + E w - \nabla_{yy} (\lambda^T f)(x, y) p \in C_2^*, \tag{1}$$
$$w^T E w \leqq 1, \tag{2}$$
$$\lambda^T e_k = 1, \tag{3}$$
$$w \in R^m, \quad \lambda \in \text{int} K^*, \quad x \in C_1. \tag{4}$$