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Global smooth solutions for the one-dimensional spin-polarized transport equation

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ABSTRACT

The existence of global smooth solutions of spin-polarized transport equation in dimension one is studied. We prove that for smooth initial data, the equation admits a unique global smooth solution without smallness restriction. When the Gilbert damping term vanishes ($\gamma = 0$), refined *a priori* estimates are made and the global existence and uniqueness of smooth solutions are also obtained.

1. Introduction

In this paper, we consider the following coupled *spin-polarized transport equation*

$$\begin{cases} \dfrac{\partial s}{\partial t} = -\partial_x J_s - s - s \times m \\ \dfrac{\partial m}{\partial t} = -m \times (\partial_x^2 m + s) + \gamma m \times \dfrac{\partial m}{\partial t}, \end{cases} \tag{1.1}$$

where (s, m) is the unknown: $s = (s_1, s_2, s_3) : \Omega \subset R^1 \to R^3$ denotes the spin accumulation, and $m = (m_1, m_2, m_3) : \Omega \subset R^1 \to \mathbb{S}^2$, the unit sphere, denotes the magnetization field, $\times$ denotes the cross product for R^3-valued vectors and finally

$$J_s = \beta(\partial_x s \cdot m)m - \partial_x s$$

for some constant $0 < \beta < 1$. This system is supplemented by initial data $s(x, 0) = s_0(x)$ and $m(x, 0) = m_0(x)$. The spin-polarized transport equation (1.1) appears in spin-magnetization system when one takes into account the diffusion process of the spin accumulation; see [1–3] for more physical background and the derivations. When $s \equiv 0$, system (1.1) reduces to the so-called Landau–Lifshitz equation or the Gilbert equation, which was proposed by Landau and Lifshitz in 1935 when studying the dispersive theory of magnetization of ferromagnets; see [4–10] and the references therein for the backgrounds and the mathematical studies. The term $m \times \frac{\partial m}{\partial t}$ is called the Gilbert damping term and hence parameter γ is called the Gilbert damping coefficient.

Eq. (1.1) is an important model in ferromagnetic theory. Indeed, recently Slonczewski [11] and Bergers [12,13] introduced a new mechanism for magnetization reversal in magnetic multilayers. In their approach, the electron spins are

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polarized, which exert additional torque on the magnetization. When applied to semiconductor devices, it could potentially revolutionize the magnetic recording industry and has been studied comprehensively in these years. However in the models introduced in [13,12,11], the spin accumulation is assumed to be uniform. To be more consistent with experiments, where spatial variations in the spin density have been found to be important, a new one-dimensional model was put forward in [2,3] taking into account the diffusion process of the spin accumulation. Later in [1], Garcia-Cervera and Wang extend the model in [3] to the higher-dimensional case.

When coupled with a quasilinear parabolic equation for the spin accumulation s, (1.1) brings new difficulties in mathematical studies and needs more delicate treatments. The only known results are the existence of global weak solutions by Garcia-Cervera and Wang by Galerkin method in [1] and the global existence and uniqueness of smooth solutions in dimension 2 under smallness condition in [14]. The local smooth solution is obtained in [14] by inverse function theory, which we state as Theorem 2.1 for reader's convenience. However to extend this local solution to globality, it is required that the initial data are small in the following sense: there exists some constant $\lambda_0 > 0$ depending on the parameters such that

$$\|s_0\|_{L^2}^2 + \|\nabla m_0\|_{L^2}^2 \le \lambda_0. \tag{1.2}$$

We would like to comment that it seems to be a challenging problem to get rid of the smallness condition.

The major and original goal of this article is to study the global existence and uniqueness for smooth initial data in dimension one under periodical setting. We solve this problem with an affirmative answer, see Theorem 2.2 for a clearer statement. However when $\gamma = 0$, the estimates leading to Theorem 2.2 fail to get the desired result. One main reason for this breakdown lies in the bad dependence of the estimates on the Gilbert damping coefficient γ: when γ goes to zero, the bound goes to infinity hence cannot be controlled. Therefore some delicate *a priori* estimates should be made to solve this problem. Our approach is by observing the fact that since m lies on the unite sphere $\mathbb{S}^2$, $\{m, m_x, m \times m_x\}$ forms an orthogonal basis in R^3 and higher order derivatives of m (in particular m_{xx}) can be expanded by this basis. This observation finally makes it possible to get the global *a priori* estimates which allow us to extend the local solution in Theorem 2.1 to globality.

This paper is organized as follows. In the subsequent section, the global existence result is obtained for $\gamma > 0$ and in Section 3, some refined estimates are made, and finally the global existence and uniqueness of smooth solutions for $\gamma = 0$ are obtained, see Theorem 3.1. As a corollary, the global existence and uniqueness of smooth solutions for the Cauchy problem on R^1 are also obtained.

Throughout this paper, the letters C and c will denote some positive constants which can vary from line to line. $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$ will denote the norm and inner product in $L^2(\Omega)$ respectively.

2. *A priori* estimates for $\gamma > 0$

In this section, we consider the case $\gamma > 0$. We will show the global existence and uniqueness of smooth solutions for (1.1) in the periodical case. For this, we set $\Omega = [-L, L]$. For clarity purpose, we rewrite

$$\partial_x((\partial_x s\cdot m)m) = (\partial_x^2 s\cdot m)m + \tilde{\partial}_x((\partial_x s\cdot m)m),$$

where $\tilde{\partial}_x((\partial_x s\cdot m)m) = (\partial_x s\cdot m)\partial_x m + (\partial_x s\cdot\partial_x m)m$. Set

$$A(m) = \begin{pmatrix} 1-\beta m_1^2 & -\beta m_1 m_2 & -\beta m_1 m_3 \\ -\beta m_2 m_1 & 1-\beta m_2^2 & -\beta m_2 m_3 \\ -\beta m_3 m_1 & -\beta m_3 m_2 & 1-\beta m_3^2 \end{pmatrix}.$$

Since $0 < \beta < 1$ is constant and $|m| = 1$, there exist two positive real numbers $0 < \lambda < \Lambda$ such that

$$\lambda|\xi|^2 \le \xi A(m)\xi^T \le \Lambda|\xi|^2, \quad \forall \xi \in R^3, \xi \ne 0.$$

Using these notations, the s-equation can be rewritten as

$$\frac{\partial s}{\partial t} - A(m)\partial_x^2 s + s = -\beta\tilde{\partial}_x((\partial_x s\cdot m)m) - s\times m. \tag{2.1}$$

On the other hand, since $m \in \mathbb{S}^2$, the m-equation can be rewritten as

$$(1+\gamma^2)\frac{\partial m}{\partial t} = -m\times(\partial_x^2 m + s) - \gamma m\times(m\times(\partial_x^2 m + s)). \tag{2.2}$$

Although system (1.1) is strongly parabolic, the local existence of smooth solutions is not trivial. One can use Hamilton's idea in [15] to derive the local existence of smooth solutions to system (1.1) with smooth initial data (s_0, m_0), see Section 2 in [14] for the details of the derivation. For reader's convenience, we cite the main result below.

Theorem 2.1. *Let the initial data $(s_0, m_0) : \Omega \to R^3 \times \mathbb{S}^2$ be a smooth map, then under the periodical boundary condition, there exists some $T^* > 0$, such that system (1.1) has a unique smooth solution on $[0, T^*]$.*