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The non-relativistic limit of Euler–Maxwell equations for two-fluid plasma

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ABSTRACT

This paper is concerned with two-fluid time-dependent non-isentropic Euler–Maxwell equations in a torus for plasmas or semiconductors. By using the method of formal asymptotic expansions, we analyze the non-relativistic limit for periodic problems with the prepared initial data. It is shown that the small parameter problems have unique solutions existing in the finite time interval where the corresponding limit problems (compressible Euler–Poisson equations) have smooth solutions. Moreover, the formal limit is rigorously justified by an iterative scheme and an analysis of asymptotic expansions up to any order.

1. Introduction

We investigate the non-relativistic limit problem for the (rescaled) two-fluid Euler–Maxwell (TEM) systems which takes the form [1–4]

$$\partial_t n_\alpha + \operatorname{div}(n_\alpha u_\alpha) = 0, \tag{1.1}$$

$$\partial_t (n_\alpha u_\alpha) + \operatorname{div}(n_\alpha u_\alpha \otimes u_\alpha) + \nabla p_\alpha(n_\alpha) = q_\alpha n_\alpha (E + \gamma u_\alpha \times B) - \frac{n_\alpha u_\alpha}{\tau_\alpha}, \tag{1.2}$$

$$\lambda^2 \partial_t E - \frac{1}{\gamma}\nabla \times B = -(q_i n_i u_i + q_e n_e u_e), \quad \partial_t B + \frac{1}{\gamma}\nabla \times E = 0, \tag{1.3}$$

$$\lambda^2 \operatorname{div} E = n_i - n_e, \quad \operatorname{div} B = 0, \tag{1.4}$$

where the index $\alpha = e, i$ and $(x, t) \in \mathcal{T}^3 \times [0, T]$. Here, n_e, u_e (respectively, n_i, u_i) denote the scaled density and mean velocity vector of the electrons (respectively, ions) and E, B the scaled electric field and magnetic field. They are functions of a three-dimensional position vector $x \in \mathcal{T}^3$ and of the time $t > 0$, where $\mathcal{T}^3 = (\frac{\mathbb{R}}{2\pi\mathbb{Z}})^3$ is the three-dimensional torus. The fields E and B are coupled to the particles through the Maxwell equations and act on the particles via the Lorentz force $E + \gamma u_\alpha \times B$. $q_e = -1$ and $q_i = 1$. $u_\alpha \otimes u_\alpha$ stands for the tensor product $u_{\alpha,j} u_{\alpha,k}$ for $j, k = 1, 2, 3$. In the system (1.1)–(1.4), $n_i - n_e = q_i n_i + q_e n_e$ and $q_i n_i u_i + q_e n_e u_e$ stand for the free charge and current densities for the particle respectively. Equations (1.1)–(1.2) are the mass and momentum balance laws respectively, while (1.3)–(1.4) are the Maxwell equations. Here the

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energy equations are replaced by the state equations of pressures $p_\alpha = p_\alpha(n_\alpha)(\alpha = e, i)$ which are supposed to be smooth and strictly increasing for $n_\alpha > 0$. i.e., the pressure functions are regular functions satisfying

$$p \in C^s(0,\infty), \qquad p'_\alpha(n_\alpha) > 0 \quad \text{for all } n_\alpha > 0, \tag{1.5}$$

with $s > \frac{3}{2} + 1$. Therefore, this model is isentropic.

Physically, τ_α stands for the momentum relaxation time, λ stands for the scaled Debye length, γ can be chosen to be proportional $\frac{1}{c}$, where c is the speed of light. These parameters can be chosen independently on each other according to the desired scaling and are small compared to the characteristic length of physical interest. Thus, regarding τ_α, λ and γ as singular perturbation parameters, we can study the limits problem in the system (1.1)–(1.4) as these parameters tend to zero. The limit $\lambda \to 0$ leads to $n_i = n_e$, which is the quasi-neutrality of the plasma. Hence, $\lambda \to 0$ is called the quasi-neutral limit. Also, $\tau_\alpha \to 0$ and $\gamma \to 0$ are physically called the zero-relaxation limit and the non-relativistic limit, respectively. Indeed, we know that the phenomenon of non-relativistic is important in many physical situation involving various nonequilibrium process. For example, important examples occur in inviscid radiation hydrodynamics [5], in quantum mechanics [6], Klein–Gordon–Maxwell system [7], and so on.

The Euler–Maxwell equations are more intricate than the Euler–Poisson equations, because of the complicated coupling of the Lorentz force. So there have been less studies on the Euler–Maxwell equations and their asymptotic analysis than the study on the Euler–Poisson equations. See [8–18] and the references therein. The first rigorously study of the Euler–Maxwell equations with extra relaxation terms is due to Chen et al. [19], where a global existence result to weak solutions in one-dimensional case is established by the fractional step Godunov scheme together with a compensated compactness argument. A local smooth solution theory for the Cauchy problem of compressible Hydrodynamic-Maxwell systems is established by J.W. Jerome (Ref. [4]) via a modification of the classical semigroup-resolvent approach of Kato. Paper [20] has just been studied for the convergence of one-fluid isentropic Euler–Maxwell system to compressible Euler–Poisson system via the non-relativistic limit. Peng and Wang [20] also prove that the combined non-relativistic and quasi-neutral limit $\gamma = \lambda^2 \to 0$ is the (one-fluid) incompressible Euler equations [21]. The justification of these limits is rigorous for smooth periodic solutions in time intervals independent of the parameters γ and λ. Recently, the two-fluid Euler–Maxwell equations are investigated in [22], where the formal asymptotic analysis is performed to derive a hierarchy of models for plasmas. Due to the two-fluid influence, in this present paper, we need to consider the interactions between electrons density n_e and ions density n_i through the Maxwell equations (1.3). Because of these effects, some key estimates in [20] have to be reconsidered, and our analysis depends heavily on the special nonlinear structure of the isentropic Euler–Maxwell models.

The aim of this article is to study the non-relativistic limit $\gamma \to 0$ by the method of asymptotic expansions in [20] and the theory in [23] to the periodic problem for the two-fluid Euler–Maxwell equations. In the case that the problems are confined in a torus, we prove the existence of smooth solutions to the problem (1.1)–(3.3) and their convergence to the solutions of the two-fluid compressible Euler–Poisson equations in a time interval independent of γ. That is, when γ is small, the solutions of two-fluid Euler–Maxwell equations and corresponding Euler–Poisson are similar. Then, we can use a Euler–Maxwell system to approximate a Euler–Poisson system. For this propose, we use the method of asymptotic expansions constructed by solving the two-fluid compressible Euler–Poisson equations and a linear curl–div system. The convergence of the expansions is achieved through the energy estimates for error equations derived from the asymptotic expansions and the Euler–Maxwell equations. Here we have to deal with some coupling and singular terms. For the variables n_α and $u_\alpha(\alpha = e, i)$, we adapt the techniques of Majda [24] for symmetrizable hyperbolic equations. For the fields E and B we observe that from the Maxwell equations (1.3) E and B satisfy the relation

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{T}^3}(\lambda^2|E|^2 + |B|^2)\mathrm{d}x = \int_{\mathcal{T}^3}(n_e u_e - n_i u_i)\cdot E\mathrm{d}x \tag{1.6}$$

to obtain uniform energy estimates for E and B with respect to x.

This paper is organized as follows. In the next section, we give some notations and basic lemmas; In Section 3, we derive formal asymptotic expansions of the problem (1.1)–(1.4) and prove the existence of the expansions. Section 4 is devoted to justify the asymptotic expansions up to any order under the condition that the initial expansions are well prepared.

2. Notations and basic lemmas

In the following, we denote by C various generic constants independent of γ, which can be different from one line to another one. $L^2(\mathcal{T}^3)$ is the space of square integral functions on $\mathcal{T}^3$ with the norm $\|\cdot\|$ or $\|\cdot\|_{L^2(\mathcal{T}^3)}$. For a nonnegative integer s, $H^s(\mathcal{T}^3)$ denotes the usual Sobolev space of function f satisfying $\partial_x^\beta f \in L^2(\mathcal{T}^3)(0 \le |\beta| \le s)$ with norm

$$\|f\|_s = \sqrt{\sum_{0\le|\beta|\le s}\|D^\beta f\|^2}, \tag{2.1}$$

here and after $\beta \in \mathbb{N}^3$, $D^\beta = \partial_x^\beta = \partial_{x_1}^{\beta_1}\partial_{x_2}^{\beta_2}\partial_{x_3}^{\beta_3}$ for $|\beta| = \beta_1+\beta_2+\beta_3$. Especially $\|\cdot\|_0 = \|\cdot\|$. Let $\mathcal{B}$ be a Banach space, $C^k([0,T];\mathcal{B})$ denotes the space of $\mathcal{B}$-valued k-times continuously differentiable functions on $[0,T]$. We can extend