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The existence of countably many positive solutions for singular multipoint boundary value problems[☆]

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ABSTRACT

In this paper, we study the existence of countably many positive solutions for a singular multipoint boundary value problem. By using fixed-point index theory and the Leggett–Williams' fixed-point theorem, sufficient conditions for the existence of countably many positive solutions are established.

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1. Introduction

This paper deals with the existence of countably many positive solutions for a singular multipoint boundary value problem (BVP for short)

$$(\phi_p(u'))'(t) + a(t)f(u(t)) = 0, \quad t \in (0, 1), \tag{1}$$

$$u'(0) - \sum_{i=1}^{m-2} \alpha_i u(\xi_i) = 0, \qquad u'(1) + \sum_{i=1}^{m-2} \alpha_i u(\eta_i) = 0, \tag{2}$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $(\phi_p)^{-1} = \phi_q$, and $\frac{1}{p} + \frac{1}{q} = 1$.

In this article, we assume that:

(H_1) $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_{m-2} < 1$, $\xi_i < \eta_i$, $\alpha_i > 0$ for $i = 1, 2, \ldots, m-2$; $\sum_{i=1}^{m-2} \alpha_i \xi_i < 1$, $\sum_{i=1}^{m-2} \alpha_i (1 - \eta_i) < 1$;

(H_2) $f \in C([0, \infty), (0, \infty))$;

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(H_3) There exists a sequence $\{t_i\}_{i=1}^{\infty}$ such that $t_{i+1} < t_i, t_1 < \frac{1}{2}, \lim_{i\to\infty} t_i = t_0, \lim_{t\to t_i} a(t) = \infty, i = 1, 2, \dots, a(t) \geq 0$ and

$$0 < \int_0^1 a(t)\mathrm{d}t < +\infty.$$

Moreover, $a(t)$ does not vanish identically on any subinterval of $[0, 1]$.

The study of multipoint boundary value problems for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [1,2]. Since then, more general nonlinear multipoint boundary value problems have been studied by many authors by using the Leray–Schauder continuation theorem, nonlinear alternative of Leray–Schauder type and coincidence degree theory; we refer the reader to [3–11] for some recent results. For example, Gupta [12] studied the existence of solutions for the generalized multipoint boundary value problem

$$x''(t) = f(t, x(t), x'(t)) + r(t), \quad 0 \leq t \leq 1,$$
$$x(0) = \sum_{i=1}^{m-2} \alpha_i x(\xi_i), \qquad x'(1) = \sum_{i=1}^{n-2} \beta_i x'(\eta_i),$$

where $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1,\ 0 < \eta_1 < \eta_2 < \cdots < \eta_{n-2} < 1,\ \alpha_i, \beta_i \in R$ and $(1 - \sum_{i=1}^{m-2} \alpha_i)(1 - \sum_{i=1}^{n-2} \beta_i) \neq 0$. Gupta established some existence results for the above BVP.

Recently, Ma et al. [13] obtained the existence of monotone positive solutions for the BVP

$$(\phi_p(u'))' + q(t)f(t, u) = 0, \quad t \in (0, 1), \tag{3}$$

$$u'(0) = \sum_{i=1}^{n} \alpha_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{n} \beta_i u(\xi_i), \tag{4}$$

where $\xi_i \in (0, 1)$ and $0 \leq \alpha_i, \beta_i < 1$ satisfy $0 \leq \sum_{i=1}^{n} \alpha_i,\ \sum_{i=1}^{n} \beta_i < 1$. The main tool is the monotone iterative technique. Wang and Hou [14] considered the multipoint BVP for the one-dimensional p-Laplacian

$$(\phi_p(u'))' + f(t, u) = 0, \quad t \in (0, 1),$$
$$\phi_p(u'(0)) = \sum_{i=1}^{n-2} a_i \phi_p(u'(\xi_i)), \qquad u(1) = \sum_{i=1}^{n-2} b_i u(\xi_i),$$

where $\xi_i \in (0, 1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$, and $a_i, b_i \in [0, \infty),\ 0 < \sum_{i=1}^{n-2} a_i < 1$, and $\sum_{i=1}^{n-2} b_i < 1$. Using a fixed-point theorem in a cone, the authors provided sufficient conditions for the existence of multiple positive solutions to the above BVP.

However, to the best of our knowledge, no work has been done for BVP (1), (2). The aim of this paper is to fill the gap in the relevant literature. By a positive solution of (1) and (2), one means a function $u(t)$ that is positive on $0 < t < 1$ and satisfies the differential equation (1) and the boundary conditions (2).

This paper is organized as follows. In Section 2, for the convenience of the reader, we give some definitions; in Section 3, we present some lemmas needed to prove our main results. Section 4 is devoted to the presentation and proof of our main results. In Section 5, we present an example of a family of functions $a(t)$ that satisfy condition (H_3).

2. Some definitions and fixed-point theorems

In this section, we present here some necessary definitions and notations.

Definition 2.1. Let E be a real Banach space and $P \subset E$ be a closed, convex set. P is a cone if the following conditions are satisfied:

(i) $\lambda x \in P$ if $\lambda \geq 0$ and $x \in P$;
(ii) if $x \in P$ and $-x \in P$, then $x = 0$.

Definition 2.2. Let E be a real Banach space and $P \subset E$ be a cone. A function $\alpha : P \to [0, +\infty)$ is called a nonnegative continuous concave functional if α is continuous and

$$\alpha(tx + (1 - t)y) \geq t\alpha(x) + (1 - t)\alpha(y)$$

for all $x, y \in P$ and $0 \leq t \leq 1$.

Let $a, b, r > 0$ be constants, $P_r = \{u \in P : \|u\| < r\}$, $P(\alpha, a, b) = \{u \in P : a \leq \alpha(u), \|u\| \leq b\}$.

Theorem 2.1 ([15]). *Let E be a Banach space and $P \subset E$ be a cone in E. Let $r > 0$ define $\Omega_r = \{u \in P : \|u\| < r\}$. Assume that $T : P \cap \overline{\Omega_r} \to P$ is completely continuous operator such that $Tu \neq u$ for $u \in \partial\Omega_r$;*

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