Nonlinear Analysis 73 (2010) 922–932

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Positive solutions for a diffusive Bazykin model in spatially heterogeneous environment☆

Wenshu Zhou

Department of Mathematics, Dalian Nationalities University, 116600, China

ARTICLE INFO

Article history:
Received 21 September 2009
Accepted 14 April 2010

MSC:
35J55
35B25
92C40

Keywords:
Bazykin model
Heterogeneous environment
Positive solution
Existence
Asymptotic behavior

ABSTRACT

In this paper, we investigate a diffusive Bazykin model in a spatially heterogeneous environment. We obtain some results on nonexistence and existence of positive solutions of the model. Moreover, the asymptotic behavior of positive solutions with respect to certain parameters is also studied.

1. Introduction

In 1974, Bazykin proposed a predator–prey model:

$$\begin{cases} \dfrac{\mathrm{d}X}{\mathrm{d}t} = kX - mX^2 - \dfrac{eXY}{1+\alpha X}, \\ \dfrac{\mathrm{d}Y}{\mathrm{d}t} = \dfrac{\mathrm{d}XY}{1+\alpha X} - cY, \end{cases} \tag{1}$$

where X and Y represent the densities of the prey and the predator, respectively, k, m, e, d, c are positive parameters describing the behavior of isolated populations and their interaction, and α is a nonnegative parameter determining the saturation of predator. In the limit case, i.e. $\alpha = 0$, system (1) becomes

$$\begin{cases} \dfrac{\mathrm{d}X}{\mathrm{d}t} = kX - mX^2 - eXY, \\ \dfrac{\mathrm{d}Y}{\mathrm{d}t} = Y(\mathrm{d}X - c), \end{cases} \tag{2}$$

which is a prey-dependent predator–prey model. We refer the reader to [1–10] and the references therein for detailed background and related studies on systems (1) and (2). In the special case where $e = m + d$, system (2) becomes

☆ Supported by National Natural Science Foundation of China (No. 10901030) and Department of Education of Liaoning Province (No. 2009A152).
E-mail address: wolfzws@163.com.

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.04.006

$$\begin{cases} \dfrac{dX}{dt} = kX\left(1 - \dfrac{X+Y}{\theta}\right) - dXY, \\ \dfrac{dY}{dt} = dXY - cY, \end{cases}$$

where $\theta = k/m$ is called the carrying capacity, which is a susceptible-infected epidemic model; see for example [11] and the references therein.

By suitable scaling changes, it follows from system (2) that

$$\begin{cases} \dfrac{du}{dt} = \lambda u - au^2 - buv := f_1(u, v), \\ \dfrac{dv}{dt} = v(u-1) := f_2(u, v), \end{cases} \tag{3}$$

where the parameters λ, a and b are positive constants. It is easy to see that system (2) has a positive steady-state if and only if $\lambda > a$, in the case, the positive steady-state $(\overline{u}, \overline{v})$ is uniquely determined by

$$(\overline{u}, \overline{v}) = \left(1, \frac{\lambda - a}{b}\right).$$

To take into account the inhomogeneous distribution of the prey and predator in different spatial locations within a fixed bounded domain at any given time, and the natural tendency of each species to diffuse to areas of smaller population concentration, we are led to the following PDE system of the reaction–diffusion type:

$$\begin{cases} u_t - d_1\Delta u = f_1(u, v) & \text{in } \Omega \times (0, +\infty), \\ v_t - d_2\Delta v = f_2(u, v) & \text{in } \Omega \times (0, +\infty), \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial\Omega \times (0, +\infty), \\ u(x, 0) = u_0 > 0, \quad v(x, 0) = v_0 \geqslant 0 & \text{in } \Omega \times (0, +\infty), \end{cases} \tag{4}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, ν is the outward unit normal vector on $\partial\Omega$, $\partial_\nu = \frac{\partial}{\partial\nu}$. The homogeneous Neumann boundary condition indicates that this system is self-contained with zero population flux across the boundary. The constants d_1 and d_2, called diffusion coefficients, are positive. It is obvious that $(\overline{u}, \overline{v})$ is the only positive constant solution of (4) if $\lambda > a$.

For sake of convenience, we assume $d_1 = d_2 = 1$. Then the stationary problem of (4) can be written as

$$\begin{cases} -\Delta u = \lambda u - au^2 - buv & \text{in } \Omega, \\ -\Delta v = v(u-1) & \text{in } \Omega, \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial\Omega, \end{cases} \tag{5}$$

where the parameters λ, a and b are positive constants. By a standard analysis, one can show that problem (5) has no non-constant positive solution (cf. [12]).

Motivated by [13–15], in the present paper, we consider problem (5) in a spatially heterogeneous environment:

$$\begin{cases} -\Delta u = \lambda u - a(x)u^2 - buv & \text{in } \Omega, \\ -\Delta v = v(u-1) & \text{in } \Omega, \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial\Omega, \end{cases} \tag{6}$$

where the parameters λ and b are positive constants, and $a(x)$ is a nonconstant, continuous function satisfying one of the following conditions:

(H1) $a(x) > 0$ on $\overline{\Omega}$.
(H2) $a(x) = 0$ on $\overline{\Omega}_0 \subset \Omega$ and $a(x) > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$, where Ω_0 is a simply connected domain with smooth boundary.

The region where $a(x)$ vanishes may be referred to as the "competition free zone" for the prey (cf. [15]).

We are concerned with existence and asymptotic behavior of positive solutions for problem (6). Before stating our results, we introduce some notations. For any $\phi \in C(\overline{\Omega})$, we denote by $\lambda_1^{\Omega}(\phi)$ and $\lambda_1^{\Omega,N}(\phi)$ the first eigenvalues of the operator $-\Delta + \phi$ in Ω with the homogeneous Dirichlet boundary condition and Neumann boundary condition, respectively. For simplicity, we write $\lambda_1^{\Omega}(0) = \lambda_1^{\Omega}$ and $\lambda_1^{\Omega,N}(0) = \lambda_1^{\Omega,N}$. It is well known that $\lambda_1^{\Omega}(\phi) > \lambda_1^{\Omega,N}(\phi)$, and $\lambda_1^{\Omega}(\phi)$ and $\lambda_1^{\Omega,N}(\phi)$ are all continuous and strict increasing in ϕ, i.e. if $\phi_1 \leqslant \phi_2$ and $\phi_1 \not\equiv \phi_2$, then

$$\lambda_1^{\Omega}(\phi_1) < \lambda_1^{\Omega}(\phi_2), \qquad \lambda_1^{\Omega,N}(\phi_1) < \lambda_1^{\Omega,N}(\phi_2).$$

Moreover, $\lambda_1^{\Omega}(\phi)$ is strictly decreasing in Ω, i.e. if $\Omega_1 \subset \Omega_2$ and $\Omega_1 \neq \Omega_2$, then $\lambda_1^{\Omega_1} > \lambda_1^{\Omega_2}$.

We will study the existence in the following three cases:

(i) (H1) holds and $\lambda > 0$;
(ii) (H2) holds and $\lambda < \lambda_1^{\Omega_0}$;
(iii) (H2) holds and $\lambda \geqslant \lambda_1^{\Omega_0}$.