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On partial synchronization of nonlinear oscillations of two Berger plates coupled by internal subdomains

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ABSTRACT

The problem of nonlinear oscillations of two Berger plates occupying bounded domains Ω in different parallel planes and coupled by internal subdomains $\Omega_1 \subset \Omega$ is considered. A dynamical system generated by the problem in the space $H = [H_0^2(\Omega)]^2 \times [L_2(\Omega)]^2$ is studied. The long-time behavior of the trajectories of the system and its dependence on the value of the coupling parameter γ is described in terms of the system global attractor. In particular, we prove a synchronization phenomenon at the level of attractor for the system. Namely, we consider a (limiting) dynamical system generated by a suitable second order in time evolution equation in the space $\tilde{H}$ consisting of the elements from H with coordinates equal for the values of the spatial variable x from the closed set $\overline{\Omega_1}$: $\tilde{H} = \{y = (y_1, y_2, y_3, y_4) \in H : y_1(x) = y_2(x),\ y_3(x) = y_4(x),\ x \in \overline{\Omega_1}\}$, and prove that the attractor of the system describing oscillations of two partially coupled Berger plates approaches the attractor of the limiting system as γ tends to the infinity.

1. Introduction

In this paper we study the problem of nonlinear oscillations of two partially coupled plates. The plates when undisturbed occupy bounded domains $\Omega \subset \mathbb{R}^2$ in different parallel planes. The boundary $\Gamma = \partial\Omega$ of the plates is clamped. The internal parts of the plates $\Omega_1 \subset \Omega$ are elastically coupled and the parts $\Omega_0 = \Omega \setminus \Omega_1$ are free. Denote by $u(x, t)$ and $v(x, t)$ the vertical with respect to the undisturbed state displacement of the point $x = (x_1, x_2) \in \Omega$ of the plates at time t. In the framework of Berger approach (see [1]) the problem can be described by the following system of partial differential equations:

$$\begin{cases} u_{tt} + \mu u_t + \Delta^2 u + \left[Q - \beta \int_\Omega |\nabla u|^2 dx\right] \Delta u + \gamma\chi(x)(u - v) = p_0, \\ v_{tt} + \mu v_t + \Delta^2 v + \left[Q - \beta \int_\Omega |\nabla v|^2 dx\right] \Delta v + \gamma\chi(x)(v - u) = p_0, \end{cases} \tag{1}$$

supplemented by the boundary conditions:

$$u = \partial_\nu u = v = \partial_\nu v = 0 \quad \text{for } x \in \Gamma, \tag{2}$$

with the initial data

$$\begin{aligned} u|_{t=0} &= u^0, \qquad v|_{t=0} = v^0, \\ u_t|_{t=0} &= u^1, \qquad v_t|_{t=0} = v^1, \end{aligned} \tag{3}$$

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where $\nu = (\nu_1, \nu_2)$ is the unit outer normal to Γ. The positive parameter $\mu > 0$ expresses the environmental resistance. The terms $\mu u_t,\ \mu v_t$ represent linear interior damping and introduce dissipation mechanism into the system. The function $p_0 = p_0(x)$ stands for external transverse load applied to the plates, the parameter $Q \in \mathbb{R}$ is proportional to the values of compressive forces acting in the planes of the plates, and β is a positive parameter. The function $\chi(x)$ from $C_0^\infty(\Omega)$ such that $\chi(x) > 0$ for $x \in \Omega_1$ and $\chi(x) = 0$ for $x \in \Omega \setminus \Omega_1$ describes the character of the plate coupling, and the non-negative parameter γ is proportional to intensity of the coupling.

The focus of the paper is on the long-time dynamics of system (1)–(3). We use the results in [2], where a complete abstract theory for a second order in time evolution equation (with, altogether, nonlinear interior damping function) is developed, to study the problem. Thus the well-posedness of system (1)–(3) in the space $H = [H_0^2(\Omega)]^2 \times [L_2(\Omega)]^2$ and the existence of a compact global attractor for the dynamical system generated by (1)–(3) in H follow from Theorem 1.5 and 3.34 in [2] respectively. Following the ideas in [2], we get that the attractor is finite dimensional and smooth (i.e. it belongs to a space more smooth than the phase spaced of the dynamical system H). The key ingredient in the proofs of these properties of attractor is a stabilizability inequality implying that the difference of two solutions can be stabilized exponentially to a compact set. Stabilizability inequalities were initially used in the context of controllability theory (see, e.g., [3]). The importance of the inequalities for studying infinite dimensional dynamics was for the first time demonstrated in [4], see also [2] and references therein. Stabilizability estimates for systems with nonlinear interior damping can be found in [5] for the case of a single Berger plate and in [6] for the case of two Berger plates coupled on the whole domain Ω, we refer to [7,8] for a stabilizability inequality for von Karman plate, and to [4,9,2] for wave equations. A specific feature of stabilizability inequalities for coupled systems (in [6] for the problem of two plates coupled on the whole domain Ω and in this paper for problem (1)–(3)) is that their constants are independent of the coupling parameter γ.

We utilize the properties of the global attractor of the dynamical system generated by (1)–(3) to study synchronization phenomena of plate oscillations. The notion "synchronization" has a long history. An account of its diversity of occurrence can be found in the book by Strogatz [10], which contains an extensive list of references. Synchronization of coupled dissipative equations has been investigated mathematically by Rodrigues [11], Afraimovich and Rodrigues [12], Kloeden [13], Caraballo and Kloeden [14]. Substantial results were obtained in the case of finite dimensional systems. In the case of infinite dimensions some results are available for parabolic systems (see [15–17]). We refer to [18] for the case of general infinite dimensional coupled system with Lipschitz nonlinearities. Synchronization phenomena for the systems consisting of several Berger plates coupled on the whole domain Ω were treated in [19,20,6].

In contrast to these works the present paper deals with synchronization phenomena of the infinite dimensional dynamics generated by the partially coupled problem (1)–(3). The structure of the function $\chi(x)$ describing the character of the plate coupling (its support is a strictly internal subdomain Ω_1 of the domain Ω) suggests that the long-time dynamics of the system is partially synchronized on the subdomain Ω_1. We say that the dynamics of a system is partially synchronized (on the domain Ω_1) if the difference of the coordinates $u(x,t)$ and $v(x,t)$ vanishes for $x \in \Omega_1$ with t approaching the infinity: $\|u(x,t) - v(x,t)\|_{H_0^2(\Omega_1)} + \|u_t(x,t) - v_t(x,t)\|_{L_2(\Omega_1)} \to 0$. The phenomenon of partial synchronization of system dynamics corresponds to a special structure of the system global attractor, which belongs to the subspace $\tilde{H}$ consisting of the points $y(x) = (y_1(x), y_2(x), y_3(x), y_4(x))$ from H with coordinates equal for x in $\overline{\Omega_1}$: $y_1(x) = y_2(x),\ y_3(x) = y_4(x),\ x \in \overline{\Omega_1}$.

The main result of the paper is that the long-time dynamics described by system (1)–(3) is partially synchronized on the domain Ω_1 (where the equations are coupled) in the limit for γ approaching the infinity. Namely, we prove that the global attractor of the system generated by (1)–(3) approaches the attractor of a dynamical system generated in $\tilde{H}$ by a suitable (limiting) problem as γ tends to the infinity.

The structure of the paper is as follows. Section 1 of the paper is an introduction. We set our main assumptions in Section 2.1 and introduce the dynamical system generated by problem (1)–(3) in Section 2.2, whereas Section 2.4 deals with the limiting dynamical system. We state the existence of the compact global attractor of the system generated by (1)–(3) and discuss its properties in Section 2.3 (see Theorem 5). The proof of the theorem is found in Section 4. Thus Section 4.1 contains the proof of dissipativity of the dynamical system generated by (1)–(3). The stress is made on the fact that the dissipativity radius can be chosen independent of the coupling parameter γ. Section 4.2 concerns stabilizability inequality and the compactness properties of the global attractor of the system. Section 3 contains the main result of the paper which states the synchronization phenomenon for problem (1)–(3) in the limit as $\gamma \to \infty$ (see Theorem 10). The proof of the main result is found in Section 5.

2. Preliminary discussion of dynamics generated by (1) and limiting system

2.1. Main assumptions

Throughout the paper the assumptions on the parameters of system (1) are:

Assumption 2.1. 1. the constant parameters are such that $\mu > 0,\ Q \in \mathbb{R},\ \beta > 0$;
2. $p_0(x) \in L_2(\Omega)$;
3. the coupling parameter γ is non-negative: $\gamma \geq 0$, and the function $\chi(x) \in C_0^\infty(\Omega)$ describing the coupling is such that $\chi(x) > 0$ for $x \in \Omega_1$ and $\chi(x) = 0$ for $x \in \Omega \setminus \Omega_1$, where Ω_1 is an open strictly internal subdomain of Ω with smooth boundary.