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Oscillation susceptibility analysis along the path of longitudinal flight equilibriums

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ABSTRACT

In this paper the oscillation susceptibility of an aircraft in a longitudinal flight with constant forward velocity is analyzed in different flight models. Conditions which ensure such a flight, and equations governing the flight are presented. The stability of the equilibriums appearing is analyzed and the existence of Hopf bifurcations and saddle–node bifurcations is researched. For two aircrafts in a simplified model it is shown that saddle–node bifurcations are present and there are no Hopf bifurcations. It is shown that for the elevator deflection there are two turning points $\underline{\delta_e} < \overline{\delta_e}$, having the property that if $\delta_e \notin \left[\underline{\delta_e}, \overline{\delta_e}\right]$, then the angle of attack α and the pitch rate q oscillate with the same period, while the pitch angle θ increases (decreases) tending to $+\infty$ $(-\infty)$. The behavior of the aircraft is simulated in the simplified model when the elevator deflection δ_e varies in the range $\left(\underline{\delta_e}, \overline{\delta_e}\right)$ and when δ_e leaves this range. For one of the aircrafts the analysis is performed also in the not simplified model, showing the differences between the results obtained in different models.

1. Introduction

Interest in oscillation susceptibility of an aircraft is generated by crashes of high performance fighter airplanes, such as the YF = 22A and B-2, due to oscillations that were not predicted during the aircraft development [1]. Flying qualities and Pilot-Induced-Oscillation (PIO) prediction are based on linear analysis and their quasi-linear extensions [2]. These analysis cannot, in general, predict the presence or the absence of PIO, because of the large variety of nonlinear pilot–aircraft interactions that have been identified as factors contributing to PIOs. Some of these factors include pilot behavioral transitions, actuator rate limiting [3–5] and changes in aircraft dynamics caused by transitions in operating conditions [6], gain scheduling and switching [7]. The analysis of nonlinear PIO involves the computation of nonlinear phenomena including bifurcations (Hopf or fold bifurcations) that lead sometimes to large changes in the stability of the Pilot–Vehicle-System (PVS).

According to [1], PIO analysis means the evaluation of the PIO potential of a given aircraft:

- Identify characteristics of the pilot–aircraft system that may result in PIO.
- Demonstrate the potential for PIO by analysis and simulations using appropriate piloting tasks and test maneuvers.
- Distinguish aircraft configurations that are less susceptible to PIOs from those that have high PIO potential.
- Suggest “fixes” to reduce and/or eliminate PIO susceptibility.

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As examples in [1] the X-15 PIO caused by the rate limiting and an F/A-18 PIO caused by nonlinear category III triggers are presented. The limit cycle amplitudes as functions of pilot gain are computed for the longitudinal flight equations of motion and large jump in limit cycle amplitude indicating a significant change in PVS stability is revealed.

Several years ago, ONERA undertook the development as a methodology, based on bifurcation theory devoted to the analysis of the asymptotic behavior of nonlinear differential equations depending on parameters. In [8] the author applies this methodology to a real combat aircraft, the German–French Alpha-Jet, and presents some significant results related to oscillatory motions and to sensitivity of spin behavior. Bifurcation theory was used in [9] for the model selection for high incidence flight mechanics analysis.

A common feature of the above presented papers is the research of the Hopf bifurcations which conduct to the oscillations of all the parameters with the same period.

Our aim in this paper is to analyze the oscillation susceptibility of an aircraft in a longitudinal flight with constant forward velocity in different flight models. For this purpose, equations governing such a flight and the conditions which ensure the existence of such a flight are presented. Using these equations the equilibriums, which appear in such a flight, are analyzed from the point of view of the presence of Hopf bifurcations or saddle–node bifurcations. For two aircrafts in a simplified model it is shown that saddle–node bifurcations are present and there are no Hopf bifurcations. It is shown that for the elevator deflection there are two turning points $\underline{\delta_e} < \overline{\delta_e}$, having the property that if $\delta_e \notin \left[\underline{\delta_e}, \overline{\delta_e}\right]$, then the angle of attack α and the pitch rate q oscillate with the same period, while the pitch angle θ increases (decreases) tending to $+\infty$ $(-\infty)$. The behavior of the aircraft is simulated when the elevator deflection δ_e is in the range $\left(\underline{\delta_e}, \overline{\delta_e}\right)$ and when δ_e leaves this range. For one of the aircrafts the analysis is performed also in the not simplified model, showing the differences between the results obtained in different models.

2. Preliminaries

The system of differential equations [10,11], which describes the motion around the center of gravity of a rigid aircraft, with respect to an xyz body-axis system, where xz is the plane of symmetry, is:

$$
\begin{cases}
\frac{\overset{\circ}{V}}{V} \cdot \cos\alpha \cdot \cos\beta - \overset{\circ}{\beta} \cdot \cos\alpha \cdot \sin\beta - \overset{\circ}{\alpha} \cdot \sin\alpha \cdot \cos\beta = r \cdot \sin\beta - q \cdot \sin\alpha \cdot \cos\beta - \frac{g}{V} \cdot \sin\theta + \frac{X}{m \cdot V} \\
\frac{\overset{\circ}{V}}{V} \cdot \sin\beta + \overset{\circ}{\beta} \cdot \cos\beta = p \cdot \sin\alpha \cdot \cos\beta - r \cdot \cos\alpha \cdot \cos\beta + \frac{g}{V} \cdot \sin\varphi \cdot \cos\theta + \frac{Y}{m \cdot V} \\
\frac{\overset{\circ}{V}}{V} \cdot \sin\alpha \cdot \cos\beta - \overset{\circ}{\beta} \cdot \sin\alpha \cdot \sin\beta + \overset{\circ}{\alpha} \cdot \cos\alpha \cdot \cos\beta \\
\quad = -p \cdot \sin\beta + q \cdot \cos\alpha \cdot \cos\beta + \frac{g}{V} \cdot \cos\varphi \cdot \cos\theta + \frac{Z}{m \cdot V} \\
I_x \cdot \overset{\circ}{p} - I_{xz} \cdot \overset{\circ}{r} = \left(I_y - I_z\right) \cdot q \cdot r + I_{xz} \cdot p \cdot q + L \\
I_y \cdot \overset{\circ}{q} = (I_z - I_x) \cdot p \cdot r - I_{xz} \cdot \left(p^2 - r^2\right) + M \\
I_z \cdot \overset{\circ}{r} - I_{xz} \cdot \overset{\circ}{p} = \left(I_x - I_y\right) \cdot p \cdot q - I_{xz} \cdot q \cdot r + N \\
\overset{\circ}{\varphi} = p + q \cdot \sin\varphi \cdot \tan\theta + r \cdot \cos\varphi \cdot \tan\theta \\
\overset{\circ}{\theta} = q \cdot \cos\varphi - r \cdot \sin\varphi \\
\overset{\circ}{\psi} = \frac{q \cdot \sin\varphi + r \cdot \cos\varphi}{\cos\theta}.
\end{cases}
\tag{1}
$$

The state parameters of this system are: forward velocity V, angle of attack α, sideslip angle β, roll rate p, pitch rate q, yaw rate r, Euler roll angle φ, Euler pitch angle θ and Euler yaw angle ψ. The constants I_x, I_y, and I_z = moments of inertia about the x-, y- and z-axis, respectively; I_{xz} = product of inertia, g = gravitational acceleration; and m = mass of the vehicle. The aerodynamical forces X, Y, Z and moments L, M, N are functions of the state parameters and the control parameters: δ_a = aileron deflection; δ_e = elevator deflection; and δ_r = rudder deflection (the body flap, speed break, δ_c, δ_{ca} are available as additional controls but, for simplicity, they are set to 0 in the analysis to follow). A longitudinal flight is defined as a flight for which $\beta = p = r = \varphi = \psi = 0$ and $\delta_a = \delta_r = 0$. Such a flight is possible if and only if $Y = L = N = 0$ for $\beta = p = r = \varphi = \psi = 0$ and $\delta_a = \delta_r = 0$.

The system of differential equations which describes the motion of the aircraft in a longitudinal flight is:

$$
\begin{cases}
\overset{\circ}{V} = g \cdot \sin(\alpha - \theta) + \frac{X}{m} \cdot \cos\alpha + \frac{Z}{m} \cdot \sin\alpha \\
\overset{\circ}{\alpha} = q + \frac{g}{V} \cdot \cos(\theta - \alpha) - \frac{X}{m \cdot V} \cdot \sin\alpha + \frac{Z}{m \cdot V} \cdot \cos\alpha \\
\overset{\circ}{q} = \frac{M}{I_y} \\
\overset{\circ}{\theta} = q.
\end{cases}
\tag{2}
$$