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Quasi-coordinates based dynamics modeling and control design for nonholonomic systems

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ABSTRACT

The paper presents dynamics modeling of nonholonomic systems in quasi-coordinates. These non-inertial coordinates are useful in motion description of constrained systems, since their selection is arbitrary and they result in a set of equations of motion in a reduced-state form. Modeling systems in quasi-coordinates may facilitate a subsequent controller design, e.g. for underactuated systems with passive wheels or when a control input is a composite quantity with respect to coordinates that describe motion. This is in contrast to most dynamics modeling which is based on generalized coordinates and Lagrange's approach. Basic disadvantages of Lagrange's approach are that it may include systems with constraints of first order and the number of unknowns that result from Lagrange's equations increases. From the control oriented modeling perspective, Lagrange's approach requires the elimination of the constraint reaction forces in order to obtain a dynamic control model. The paper proposes an approach to control oriented modeling based on the generalization of the Boltzmann–Hamel equations. These are the generalized programmed motion equations in quasi-coordinates. This dynamics framework yields equations of motion of a constrained system in a reduced-state form, from which the dynamic control model directly follows. The framework applies to fully actuated and underactuated systems, it is computationally efficient and may facilitate a subsequent controller design.

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1. Introduction

Systems with nonholonomic constraints (NC), which we refer to as nonholonomic systems (NS) are usually included in Lagrange's approach by adding Lagrange multipliers and then eliminating them from the final set of equations of motion. These final equations are in the so called reduced-state form and they are used to develop a dynamic control model for a system by adding control inputs [12,19]. A few papers consider dynamics modeling using different approaches, e.g. in [17,18] Kane's equations are used to develop a dynamic control model of a mobile manipulator. In [2] the Boltzmann–Hamel equations are modified to facilitate modeling manipulator systems as well as wheeled vehicles. In [11] these equations are used to develop a helicopter control system. In [15] a ski-steering wheeled mining vehicle is modeled by the Boltzmann–Hamel equations. Both Lagrange's and Kane's, and the Boltzmann–Hamel equations serve for systems with first order constraints and relations between quasi-velocities, and generalized velocities are all linear there.

NS are usually meant as systems with constraints that come from the rolling contact or from the conservation law for space vehicles and manipulators [12]. Then, from the point of view of mechanics and derivation of equations of motion for them, they belong to the same class of first order NS. They may be approached by Lagrange's equations with multipliers and these equations are used to generate dynamic control models for them most often, e.g. [1,10,12,19].

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From the control theory perspective, a constraint concept, constraint sources and their classification are viewed in a different way than in analytical mechanics [4–7]. The reasons are as follows. Robotic systems, which are a representative class of mechanical systems, are designed to perform work. Usually, this is the end-effector at which work is performed. It can be writing, painting, scribing, grinding, carrying objects or other tasks. It is natural then to focus on the end-effector of a robot and specify tasks in terms of desired motions of the end-effector. Also, during the robot motion its other parts may perform specified motions or be constrained. Specifically, the end-effector or a wheeled robot may move along a specified trajectory or have some desired velocity. For wheeled robots other constraints, like the one that specifies an allowable value of the acceleration that prevents a robot wheel from slippage and mechanical shock during motion, have to be taken into account when a control strategy is designed [8]. A new group of constraints, which are tasks that can be specified by equations may be distinguished [4–7]. The source of these constraints is not in other bodies. They are non-material constraints put upon systems in order to specify tasks they have to perform. They are referred to as programmed constraints [4]. Then, the constraints on mechanical systems can be divided as follows [6]:

1. Material constraints, which are position or kinematic, which can be nonholonomic, and conservation laws.
2. Programmed constraints, which are non-material.
3. Constraints that may come from dynamic, design, control or operation specifications including underactuated systems.
4. Constraints that may specify obstacles in a robot or manipulator workspace.

The extended constraint concept and the classification of constraints that follows cause the constrained system modeling to require new approaches.

Material constraints, especially nonholonomic, are present in many system models, e.g. in wheeled system models, and the conservation law applies to space vehicles and manipulators. Dynamic models for these systems are developed based on Lagrange's or Kane's approaches as mentioned before. It means that only first order constraints are merged into these models. Programmed constraints and constraints from groups 3 and 4 are taken into account when control strategies are to be designed to obtain motions that satisfy these constraints. The above modeling procedure and control design are typical ways in which tasks are executed within nonlinear control theory framework. In [4–7] the programmed and dynamic constraints are included in the generalized programmed motion equations (GPME) derived in generalized coordinates.

From the point of view of nonlinear control theory, NS differ and may not be approached by the same control strategies and algorithms. For instance, some of them may be controlled at the kinematic level and the other may be controlled at the dynamic level only [3,6,9,16]. Their control properties depend upon the way they are designed and propelled, i.e. how many control inputs are available and whether their wheels, if there are any, are powered or not. They are divided into two control groups, which are treated separately, i.e. the group of fully actuated and the group of underactuated NS, see e.g. [6,16].

A control design process consists of three main steps, i.e. a model building, a controller design and a controller implementation. Most modifications and improvements concern the third step of this process. Main motivations to the research we present are to unify the NS modeling and a subsequent controller design.

The paper deals with the first two steps of the controller design process and it presents a modeling framework that is model-based and control oriented. It is not sensitive to the NS design, and it facilitates the second step, i.e. the dynamic controller design. Thus, the dynamic modeling framework serves a unification of the NS modeling with no regard to whether a specific NS is fully actuated, underactuated, or constrained by additional constraints.

The contribution of the paper is two-folded. Firstly, we develop the model-based control oriented framework for the NS formulated in quasi-coordinates. We assume that relations between generalized velocities and quasi-velocities may be nonlinear. Also, the NS may be constrained by the NC of high order that originates from any of four groups. Secondly, we demonstrate that the formulation is unified in the sense that it is suitable for any NS modeling and, based on this formulation, a controller for the NS may be designed.

Based on examples of the NS we demonstrate how to apply this modeling framework to obtain dynamic control models and how to reuse it to design tracking control algorithms. Specifically, we select one fully actuated NS which is a 2-wheeled mobile robot, one underactuated NS, which is a roller-racer [2,7]. We also take a holonomic system, which is a two-link planar manipulator model. We make it a NS by constraining its motion in some predefined way, and underactuated by assuming a failure of one of its actuators. We demonstrate that for all these systems, one modeling framework in quasi-coordinates may be applied.

The paper is organized as follows. In Section 2 we present the theoretical background for the development of equations of motion in quasi-ccordinates. Section 3 develops the generalized Boltzmann–Hamel equations. The generalized programmed motion equations (GPME) in quasi-coordinates are derived in Section 4. Section 5 details examples. Conclusions and the list of references close the paper.

2. Concepts of quasi-coordinates and quasi-velocities

Quasi-coordinates and quasi-velocities were introduced first to derive equations of motion referred to as the Boltzmann–Hamel equations, for details see for example [13,14]. Relations between the generalized velocities and quasi-velocities were assumed to be linear and non-integrable, i.e.

$$\omega_r = \omega_r(t, q_\sigma, \dot{q}_\sigma). \quad \sigma, r = 1, \ldots, n. \tag{1}$$

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