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Decision making in multiobjective optimization aided by the multicriteria tournament decision method

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ABSTRACT

This paper proposes a new method for multicriteria analysis, named Multicriteria Tournament Decision (MTD). It provides the ranking of alternatives from best to worst, according to the preferences of a human decision-maker (DM). It has some positive aspects such as: it has a simple algorithm with intuitive appeal; it involves few input parameters (just the importance weight of each criterion).

The helpfulness of MTD is demonstrated by using it to select the final solution of multiobjective optimization problems in an *a posteriori* decision making approach. Having at hand a discrete approximation of the Pareto front (provided by a multiobjective evolutionary search algorithm), the choice of the preferred Pareto-optimal solution is performed using MTD.

A simple method, named Gain Analysis method (GAM), for verifying the existence of a better solution (a solution associated to higher marginal rates of return) than the one originally chosen by the DM, is also introduced here. The usefulness of MTD and GAM methods is confirmed by the suitable results shown in this paper.

1. Introduction

The final solution for a multiobjective optimization problem (MOP) must satisfy two conditions: it must be a Pareto-optimal point; it must be the most suitable solution, according to the preferences of a decision-maker (DM), considering multiple decision criteria. Thus, it is necessary to utilize an optimization algorithm and a decision technique for solving a MOP.

The Multicriteria Analysis studies manners of aiding man to make decisions, considering multiple criteria. There are various structures for capturing the DM's preferences: multiattribute utility theory [1], Analytic Hierarchy Process (AHP) [2], outranking relations [3] and fuzzy preference relations [4] are some of the most popular ones. In several works, such structures are coupled to optimization algorithms for solving MOPs, following three possible approaches: the *a priori*, the progressive and the *a posteriori* decision [5]. In the *a priori* decision, the DM is consulted once, before starting the search process. His preferences guide the search until the convergence to a unique point or to a reduced set of nondominated points. In the progressive decision, the DM is requested several times in the course of an iterative optimization process. The obtained information is utilized to guide the search during the subsequent iterations, until another request. The whole process is interrupted only when the DM is totally satisfied with the current solution. In the *a posteriori* decision, a multiobjective optimization algorithm meets a discrete representation of the Pareto front. Then, the DM may utilize a decision method to choose the final solution.

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Basically, the main advantage of the *a posteriori* decision approach lies in the fact that, as the search and the decision processes are segregated, it is possible to navigate through the Pareto front, considering several decision scenarios, generated by the opinion of different experts and/or by the inclusion of new criteria, without the need of executing the optimization algorithm more than once. The analysis of different decision scenarios may be so helpful that data mining tools to aid the *a posteriori* decision have been developed [6]. On the other hand, the main disadvantage of the *a posteriori* decision approach is in the high computational effort required by the search for a discrete approximation of the Pareto front. Indeed, the processing time may become unfeasible when high dimensional problems are considered.

This paper proposes a method for multicriteria analysis, the Multicriteria Tournament Decision (MTD), which provides the ranking of all alternatives, from best to worst, based on the DM's preferences. It has some positive aspects: its algorithm is simple, has intuitive appeal and few input parameters; its results are coherent and satisfactory. In order to show the MTD utility, it is applied to choose the final solution for MOPs, in an *a posteriori* decision approach.

Some researchers agree that *knee regions* in the Pareto front may correspond to attractive final solutions. A knee region consists of points for which an improvement in an objective is associated to a severe degradation in another, i.e, solutions associated to high marginal rates of return. There are multiobjective algorithms that prioritize the search for such points [7–10]. It is not the case of the Elitist Nondominated Sorting Genetic Algorithm (NSGAII), which we chose (at the expense of a higher computational cost) to search for a discrete approximation of the Pareto front [11]. It is known that human preference has imprecise and complex elements that may lead (in a justified manner) the choice to points located out of knee regions. Many multicriteria decision methods (for instance: AHP [2], Promethee II [12], Smarts [13]) do not prioritize such alternatives, as a rule. For that reason and also because, in the *a posteriori* decision, the inclusion of new criteria may generate additional solutions with high marginal rates of return, we preferred to avoid directing the search toward any specific zone of the Pareto front. In this way, the current approach allows the DM to prefer any alternative along the Pareto front. If his choice does not lie in a knee region, he is not enforced to drastically adapt his preferences. But, the Gain Analysis Method (GAM), a simple method also proposed here, can be used to verify if a slight change in the DM's preferences would result in an overall improvement of the original choice, without significant losses in any criterion.

Therefore, the *a posteriori* decision approach implemented here begins with the execution of NSGAII, in order to search for Pareto-optimal points. With a discrete approximation of the Pareto front at hand, the preferred solution is chosen using MTD. Finally, GAM verifies the existence of an alternative in the neighborhood of the previously chosen solution, but with higher marginal rates of return. It is worth mentioning that GAM can be utilized in the *a posteriori* decision approach, after the execution of MTD, or other multicriteria decision methods.

2. Problem formulation

The decision problem considered here consists in choosing the preferred Pareto-optimal solution for a MOP stated as:

$$\min F(\vec{x}) = \{f_1(\vec{x}), \ldots, f_m(\vec{x})\}, \quad \vec{x} \in L, \tag{1}$$

where L is a feasible region in $\Re^n$.

The final choice involves the following basic elements [5]:

- **The set *A* of alternatives.** This set consists of all Pareto-optimal points. It is often infinite, limited only by mathematical constraints, being a subset of $\Re^n$. But, after the execution of a multiobjective search, it is reduced to a discrete approximation of the Pareto front, i.e, a discrete set of nondominated solutions.
- **The set of criteria $\boldsymbol{C} = \{\boldsymbol{c_1}, \ldots, \boldsymbol{c_q}\}$.** Each criterion represents a viewpoint, according to which the alternatives are evaluated and compared, based on their respective attributes. Usually, it is mathematically modeled by a function $c_i(\vec{x}) : A \rightarrow \Re$ that assigns a grade to each alternative, reflecting the decision-maker's preferences. In the continuous problems, each criterion is often derived from an objective function. But, it is worth mentioning that additional criteria that do not correspond to any objective function can also be considered.

In order to simplify the description of MTD and GAM methods, it is assumed along the text that each criterion is derived from an objective function. Hence, the number of criteria and of objective functions will be considered the same: $m = q$.

3. MTD

MTD assumes the existence of a function $R(.)$, capable of reflecting the DM global interests. In order to construct it, first, each possible solution is compared with the others, considering only the ith-criterion. The pairwise comparisons are implemented through the tournament function $T_i(a, A)$, which counts the ratio of times the alternative a wins the tournament against each other b solution from A. In the case of minimization problems, the winner of each tournament (i.e., the preferred solution in the pairwise tournament) is the alternative with lower objective function value. Hence, given