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Layers of noncooperative games[☆,☆☆]

Jewgeni H. Dshalalow [*], Hao-Jan Ke

Department of Mathematical Sciences, College of Science, Florida Institute of Technology, Melbourne, FL 32901, USA

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ABSTRACT

We model and analyze classes of antagonistic stochastic games of two players. The actions of the players are formalized by marked point processes recording the cumulative damage to the players at any moment of time. The processes evolve until one of the processes crosses its fixed preassigned threshold of tolerance. Once the threshold is reached or exceeded at some point of the time (*exit time*), the associated player is *ruined*. Both stochastic processes are being "observed" by a third party point stochastic process, over which the information regarding the status of both players is obtained. We succeed in these goals by arriving at closed form joint functionals of the named elements and processes. Furthermore, we also look into the game more closely by introducing an intermediate threshold (see a *layer*), which a losing player is to cross prior to his ruin, in order to analyze the game more scrupulously and see what makes the player lose the game.

1. Introduction

The present article models and analyzes classes of stochastic games of two players with totally opposite interests. The actions of players A and B are formalized by marked point processes $\mathcal{A}$ and $\mathcal{B}$ recording the cumulative damages to A and B at any moment of time. The processes evolve until one of them crosses its fixed preassigned threshold of tolerance, M or N, respectively. Once the threshold is reached or exceeded at some point in time (*first passage time*, also referred to as *exit time*), the associated player is said to be *ruined*. Both stochastic processes are being "observed" by a third party point stochastic process $\mathcal{T}$ over which the information regarding the status of both players is obtained. This is a more realistic scenario than real time information. The former may cause some delay, but the observation frequencies can be arbitrarily refined, at least from the standpoint of our analytical model.

In a basic model, which is treated in Section 2, we are interested in the ruin time of a selected player upon nearest observation epoch, and the status of the system at this time (along with many other parameters introduced in the upcoming sections). Unlike the recent work by the first author [3,4] with time dependent analysis (and thus more detailed information about the processes at any moment of time), the present modeling and analysis will be concerned with the information for the reference times only. They are: the exit time of player A (considered as a loser, without loss of generality) from the game

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* Corresponding author. Tel.: +1 321 951 8306; fax: +1 321 723 8285.
E-mail addresses: eugene@fit.edu, edshalalow@cfl.rr.com (J.H. Dshalalow), hke@fit.edu (H.-J. Ke).

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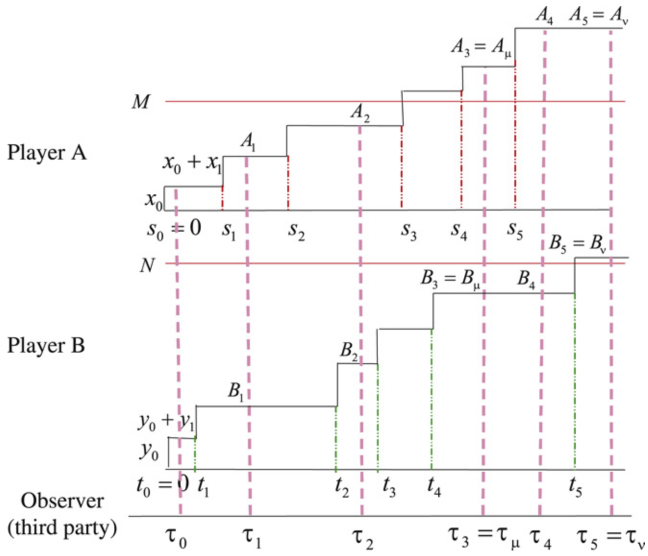

Fig. 1.

and the *pre-exit time*, i.e. an observation prior to the exit time. It is very common and quite reasonable for an observer or user to associate himself with the losing player A (rather than the winner B) to evaluate and prognosticate the worst outcome and to take measures ahead of time. The advantage of this more rudimentary model (by the way, not being a special case of [3,4]) lies in very compact explicit formulas leading to more tame computations. Some of the past work on related techniques was initiated in [2] by the first author and it was merely applied to the stock market.

In a more advanced model of Section 3, we will emulate some of the time dependence of [3,4] in the following sense. We will embed an "intermediate threshold", say $M_1(< M)$ and see, through this "layer", along with the rest of the information, when the overall damage to player A will exceed M_1 before it will exceed M. In other words, this refinement will more closely analyze the behavior of the processes prior to player's A defeat. Along these lines, we will establish an important "insensitivity" property of the processes which will lead to closed form related functionals.

Although the present paper does not generalize the recent work by the first author [3,4], it augments it to some degree and it represents an almost alternative approach to the analysis of this class of stochastic games.

Game-theoretical work is mostly devoted to economics, although it stemmed from warfare during WWII. The literature on games is vast and a part of it is that on cooperative games, like [6], while other parts belong to the class of noncooperative (antagonistic) games. Noncooperative games [1,5,10] have also been widely used in economics [7] and some warfare [11]. The main technique we are using in this paper falls into the area of fluctuation analysis related to the random walk and occurring economics [2,8], physics [9], and other areas of engineering and technology. Article [3] by the first author contains a more detailed bibliography regarding fluctuations and games.

2. A basic model

We will start with a basic antagonistic game of two players called "A" and "B".

Let $(\Omega, \mathcal{F}(\Omega), P)$ be a probability space and let $\mathcal{F}_A, \mathcal{F}_B, \mathcal{F}_\tau \subseteq \mathcal{F}(\Omega)$ be independent σ-subalgebras. Suppose

$$\mathcal{A} := \sum_{j\geq 0} x_j \varepsilon_{s_j}, \quad 0 = s_0 < s_1 < \cdots, \text{ a.s.} \tag{2.1a}$$

$$\mathcal{B} := \sum_{k\geq 0} y_k \varepsilon_{t_k}, \quad 0 < t_0 < t_1 < \cdots, \text{ a.s.} \tag{2.1b}$$

are $\mathcal{F}_A$-measurable and $\mathcal{F}_B$-measurable marked Poisson processes (ε_a is a point mass at a) with respective intensities λ_A and λ_B and position independent marking. They will represent the actions of players A and B. Player A will be attacked at times $s_1, s_2, \ldots$ and sustain respective damages of magnitudes $x_1, x_2, \ldots$ formalized by process $\mathcal{A}$. The attacks to player B are described by process $\mathcal{B}$ similarly. See Fig. 1: The processes $\mathcal{A}$ and $\mathcal{B}$ are specified by their transforms

$$E\mathrm{e}^{-u\mathcal{A}(\cdot)} = \mathrm{e}^{\lambda_A|\cdot|[g(u)-1]}, \qquad g(u) = E\mathrm{e}^{-ux_1}, \quad \mathrm{Re}(u) \geq 0, \tag{2.2}$$

$$E\mathrm{e}^{-u\mathcal{B}(\cdot)} = \mathrm{e}^{\lambda_B|\cdot|[h(u)-1]}, \qquad h(u) = E\mathrm{e}^{-uy_1}, \quad \mathrm{Re}(u) \geq 0, \tag{2.3}$$

$|\cdot|$ is the Borel–Lebesgue measure and x_j and y_k are nonnegative r.v.'s.