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Stability analysis for point delay fractional description models via linear matrix inequalities

M. De la Sen

Institute of Research and Development of Processes, Department of Electricity and Electronics, Faculty of Science and Technology, Leioa (Bizkaia), Aptdo. 644-Bilbao, 48080-Bilbao, Spain

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ABSTRACT

This paper is devoted to linear parameter systems under linear fractional representations (LFR) of parameter-dependent nonlinear systems with real–rational nonlinearities and point-delayed dynamics. The robust global asymptotic stability of the system either independent of or dependent on the delay sizes is investigated. The associate matrix inequalities are related to the time-derivatives of appropriate Lyapunov functions at all the vertices of the polytope which contains the parameterized uncertainties.

1. Introduction

Time-delay systems are very common in nature, for instance they are related to transportation problems, population growth and signal transmission methods (see, for instance, [1–8] and references therein). The stability and stabilization of those systems have been studied in the literature in connection, for instance, with the Lyapunov theory (see, for instance, [1–5,19]) or frequency domain techniques (see, for instance, [2,7,8] and references therein). Some of the related results are referred to either as being independent of the sizes of the delays or as dependent of those sizes. Within this last class of results, those related to the characterization of the first interval of admissible delay sizes allowing stabilization, merit special attention. On the other hand, the most involved group of results to obtain is that related to internal delays (i.e. in the state) since its associate dynamics possess infinitely many modes in general, [2]. Continuous-time and discrete-time systems involving delays have also been studied in [20–24]. In particular, semilinear partial functional differential equations with infinite delays have been recently investigated in [20]. In [21], previous results concerning necessary and sufficient conditions for global asymptotic stability of scalar difference equations with a single delay have been extended to second order systems by using a general treatment for Difference Equations. See, for instance, [13,25] for general related results. Periodic and non-oscillatory solutions of time-delay systems were investigated in [22,23], respectively. In [23], the impulsive case has been considered. In this paper, we consider a parameter-dependent (in general, nonlinear and time-varying) system subject to a finite set of point delays which may be, in general, defined by real–rational nonlinearities, whose parameter vector H_∞ is restricted to lie in a polytope $\boldsymbol{\Theta} \in \mathbf{R}^n$ containing the origin, This is named the so-called polytopic-delayed system following the nomenclature used for delay-free systems in [11,17]. As proposed in [11,17] for a delay-free system, the results developed in the following might be still applied if the set $\boldsymbol{\Theta}$ is not a polytope after replacing it by some polytope $\boldsymbol{\Theta}_{\mathbf{poly}} \supset \boldsymbol{\Theta}$ (see also [9,10,14,16]). The main arguments used to develop the formalism are based on the fact that the polytope where the parameters belong to defines affine function matrices of vertices which may be calculated from those of the original polytope $\boldsymbol{\Theta} \in \mathbf{R}^n$ of parameterized uncertainties. The quadratic stability of the so-called delay-free polytopic system has been investigated in [9–11]. The advantage of parameterizing uncertainties within polytopes is that conditions such as for instance, slow time variation of the parameters are not required to investigate the stability, [16,17]. The existence of Lyapunov functional candidates for the dynamic system is investigated through the derivation of matrix inequalities

E-mail address: wepdepam@lg.ehu.es.

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associated with the system and the Lyapunov functional candidate, [12,15,16,18]. Some related standard results have been provided in [24]. In the following, the robust global asymptotic stability of such a polytopic-type-delayed system subject to point delays is investigated via the Lyapunov theory.

1.1. Notation

$\mathbf{R}^{m\times n}$ $\left(\mathbf{C}^{m\times n}\right)$ is the set of real (complex) $m \times n$ matrices and $P = P^{\mathrm{T}} > 0$ stands for a real symmetric positive-definite matrix.

- For a given set S, one defines $\sigma S = \{\sigma s : s \in S\}$ if σ is a positive number.
- The convex hull of complex $m \times n$ matrices, $\vartheta_i \in \mathbf{C}^{m\times n}$ is

$$\mathbf{Co}\,\{\vartheta_1\vartheta_2,\ldots,\vartheta_l\} = \left\{\vartheta : \vartheta = \sum_{i=1}^{l}\lambda_i\vartheta_i, \sum_{i=1}^{l}\lambda_i = 1, \lambda_i \geq 0\right\}$$

- I_m is the m-identity matrix with the subscript being omitted if its size follows directly from context.

If $\mathbf{\Theta}$ is a polytope containing the origin and $\Delta_i(\theta)$ $\left(i = \overline{0,r}\right)$, $r \geq 0$ being an integer, are real-valued rational matrix functions of any order of θ then $\mathbf{\Delta}_i = \{\Delta_i(\theta)\,\theta \in \mathbf{\Theta}\}$ and $\mathbf{\Delta} = \mathbf{\Delta}_0 \times \mathbf{\Delta}_1 \times \cdots \times \mathbf{\Delta}_r$ are polytopes of υ_i vertices $\Delta_i^{(k_i)}$, $k_i = \overline{0,\upsilon_i}$; $i = \overline{0,r}$; and $\Delta^{(k_0,k_1\ldots,k_r)} = \Delta_0^{(k_0)} \times \cdots \times \Delta_r^{(k_r)}$, respectively, where '$\times$' denotes the Cartesian product of matrices (considered as sets). In our context, $\mathbf{\Theta}$ is the polytope where the system parameters belong to while $\mathbf{\Delta}_i$ is the polytope where the rational matrix function $A_i(\theta)$, defining the dynamics of the ith delay $h_i(A_0(\theta))$ describes the delay-free dynamics; (i.e. $h_0 = 0$) as the parameter vector θ runs over $\mathbf{\Theta}$; $i = \overline{0,r}$.

2. Linear fractional descriptions

Consider the parameter-dependent system subject to r point delays h_i $\left(i = \overline{1,r}\right)$:

$$\dot{x}(t) = \sum_{i=0}^{r} A_i(\theta(t))\,x(t-h_i) + B(\theta(t))\,u(t) \tag{1a}$$

$$y(t) = C(\theta(t))\,x(t) + D(\theta(t))\,u(t) \tag{1b}$$

where $h_0 = 0$, $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^{n_u}$, $y(t) \in \mathbf{R}^{n_y}$ are the state, input and measurable output signals, respectively, and A_i $\left(i = \overline{0,r}\right)$, B, C and D are real-valued rational functions of time-varying parameter vector $\theta(t)$ with $\theta \in \Theta$ for all $t \geq 0$ with $\theta(t) = (\theta_1(t), \theta_2(t), \ldots, \theta_m(t))^{\mathrm{T}}$ such that the real vector associated function is defined in such a way that (1) has a mild solution on $[0, t)$ for all $t \geq 0$ for any absolutely continuous function $\varphi : [-h, 0] \to \mathbf{R}^n$ of initial conditions $x(t) \equiv \varphi(t)$, $t \in [-h, 0]$ with $h = \mathrm{Max}_{1\leq i\leq r}(h_i)$. One defines:

- The unforced system (1) is robustly globally asymptotically stable if $\|x(t)\|$ is uniformly bounded and $\lim_{t\to\infty} x(t) = 0$ if $u \equiv 0$ for any bounded x(0). The system (1) is robustly stabilizable if there exists an output-feedback realizable control law $u(t) = K(y, \theta(t), t)$ such that the closed-loop system is robustly globally asymptotically stable. For terminology simplicity, since no confusion is expected, we refer in the sequel to robust global asymptotic stability simply as "robust stability".

- The robust stability (stabilizability) margin of (1) for an uncertainty set is $\sigma_m(\rho_m) = \mathrm{Sup}\,\{\sigma(\rho)\}$: System (1) is robustly stable (stabilizable) over $\gamma\mathbf{\Theta}$ for all $\gamma \in [0, \sigma]$ $(\gamma \in [0, \rho])$. Now, first consider the unforced version of (1) given by

$$\dot{x}(t) = \sum_{i=0}^{r} A_i(\theta(t))\,x(t-h_i)\,; \qquad y(t) = C(\theta(t))\,x(t)\,. \tag{2}$$

First LFR: Since $A_i(\theta(t))$ $\left(i = \overline{0,r}\right)$ is a real-valued rational matrix function of $\theta(t)$, the LFR description of each matrix function $A_i(\theta(t))$ exists for some appropriate matrices A_{oi}, B_{qi}, D_{pqi} $\left(i = \overline{0,r}\right)$:

$$A_i(\theta(t)) = A_{0i} + B_{qi}\Delta_i(\theta(t))\left(I_{d_i} - D_{pqi}\Delta_i(\theta(t))\right)^{-1}C_{pi} \tag{3}$$

for any $\Delta_i(\theta(t))$ such that the well-posedness condition Det $\left(I_{d_i} - D_{pqi}\Delta_i(\theta(t))\right) \neq 0, \forall\theta \in \mathbf{\Theta}$, all $t \geq 0$ where I_{d_i} is the d_i identity matrix $\left(i = \overline{0,r}\right)$. In the following, the explicit dependence of $\theta(t)$ on time is omitted in the notation for the shake of simplicity when no confusion is expected. If (2) is quadratically stable then A_{0i} $\left(i = \overline{0,r}\right)$ are strictly Hurwitzian (i.e. with all their eigenvalues in Re s < 0). A state-space realization of the state evolution of the dynamical system (2) using (3) is

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=0}^{r}\left(A_{0i}x(t-h_i) + B_{qi}q_i(t-h_i)\right)\\
p_i(t) &= C_{pi}x(t) + D_{pqi}q_i(t) = \left(I - D_{pqi}\Delta_i(\theta)\right)^{-1}C_{pi}x(t)\\
q_i(t) &= \Delta_i(\theta)\,p_i(t) = \Delta_i(\theta)\left(I - D_{pqi}\Delta_i(\theta)\right)^{-1}C_{pi}x(t)\\
\Delta_i(\theta) &= \mathrm{Diag}\left(\theta_1 I_{s_{1i}}, \ldots, \theta_m I_{s_{mi}}\right)
\end{aligned} \tag{4}$$