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Triple positive solutions of m-point BVPs for p-Laplacian dynamic equations on time scales[☆]

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Abstract

This paper is concerned with the existence of positive solutions of p-Laplacian dynamic equation $(\varphi_p(u^{\Delta}(t)))^{\nabla} + a_1(t)f(u(t)) = 0$ subject to boundary conditions $u(0) - B_0(\sum_{i=1}^{m-2} a_i u^{\Delta}(\xi_i)) = 0, u^{\Delta}(T) = 0$ or $u^{\Delta}(0) = 0, u(T) + B_1(\sum_{i=1}^{m-2} b_i u^{\Delta}(\xi_i)) = 0$, where $\varphi_p(v) = |v|^{p-2}v$ with $p > 1$. By using the five functionals fixed-point theorem, we prove that the boundary value problem has at least three positive solutions. As an application, an example is given to illustrate the result.

MSC: 34B15; 39A10

Keywords: Time scales; Boundary value problem; Positive solutions; p-Laplacian; Fixed-point theorem

1. Introduction

The development of the theory of time scales was initiated by Hilger in his Ph.D. thesis in 1988 as a theory can contain both differential and difference calculus in a consistent way. Since then, we have witnessed a lot of efforts in the field of time scales, especially in unifying the theory of differential equations in the continuous case and the theory of finite difference equations in the discrete case, see [3,4,9,14,16] and the references therein. Many works are concerned with the existence of positive solutions of boundary value problems with p-Laplacian in the continuous case, see [8,10,12,13]. However, very little work has been done to the existence of positive solutions of p-Laplacian dynamic equation on time scales, see [6,15,17], which motivate us to consider one-dimensional p-Laplacian boundary value problem on time scales.

Throughout the remainder of the paper, let $\mathbb{T}$ be a closed nonempty subset of $\mathbb{R}$, and let $\mathbb{T}$ have the subspace topology inherited from the Euclidean topology on $\mathbb{R}$. In some of the current literature, $\mathbb{T}$ is called *a time scale*. For convenience, we make the blanket assumption that $0, T$ are points in $\mathbb{T}$, for an interval $(0, T)_{\mathbb{T}}$, we always mean $(0, T) \cap \mathbb{T}$. Other types of intervals are defined similarly.

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We denote the p-Laplacian operator by $\varphi_p(v)$, i.e. $\varphi_p(v) = |v|^{p-2} v$, $p > 1$, $(\varphi_p)^{-1} = \varphi_q$ and $\frac{1}{p} + \frac{1}{q} = 1$.

For the two-point boundary value problem

$$(\varphi_p(u^{\Delta}(t)))^{\Delta} + a_1(t) f(u^{\sigma}(t)) = 0, \quad t \in [a, b]_{\mathbb{T}},$$
$$u(a) - B_3(u^{\Delta}(a)) = 0, \quad u^{\Delta}(\sigma(b)) = 0.$$

Sun and Li [15] established the existence theory for positive solutions by using some fixed-point theorems [1,5,7].

For the three-point boundary value problems

$$(\varphi_p(u^{\Delta}(t)))^{\nabla} + a_1(t) f(u(t)) = 0, \quad t \in [0, T]_{\mathbb{T}}, \tag{1.1}$$

satisfying the boundary conditions

$$u(0) - B_0(u^{\Delta}(\eta)) = 0, \qquad u^{\Delta}(T) = 0, \tag{1.2}$$

or

$$u^{\Delta}(0) = 0, \qquad u(T) + B_1(u^{\Delta}(\eta)) = 0, \tag{1.3}$$

where $\eta \in (0, \rho(T))_{\mathbb{T}}$, B_0 and B_1 satisfy

$$Bx \le B_i(x) \le Ax, \quad x \in \mathbb{R}, \ i = 0, 1, \tag{1.4}$$

here B and A are nonnegative numbers. Under some assumptions on f, by using the fixed-point theorem due to Avery and Henderson [1], He [6] proved that the boundary value problems (1.1) and (1.2) or (1.3) has at least *two* positive solutions. In particular, if $\mathbb{T} = \mathbb{Z}$, then the similar result was obtained by Liu and Ge [11]. In addition, Wang [17] used the Leggett–Williams fixed-point theorem [7] and obtained the existence criteria for at least *three* positive solutions of boundary value problems (1.1) and (1.2) or (1.3).

Motivated by [6,15,17], it is natural to consider the existence of positive solutions of problem (1.1) subject to multi-point boundary conditions. In this paper, using a new fixed-point theorem different from fixed-point theorems used in [6,11,15,17], we consider boundary value problem (1.1) satisfying the boundary conditions

$$u(0) - B_0\left(\sum_{i=1}^{m-2} a_i u^{\Delta}(\xi_i)\right) = 0, \qquad u^{\Delta}(T) = 0, \tag{1.5}$$

or

$$u^{\Delta}(0) = 0, \qquad u(T) + B_1\left(\sum_{i=1}^{m-2} b_i u^{\Delta}(\xi_i)\right) = 0, \tag{1.6}$$

where $\xi_i \in (0, T)_{\mathbb{T}}$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < T$ and $a_i, b_i \in [0, \infty)(i = 1, 2, \ldots, m-2)$. By using the five functionals fixed-point theorem in a cone [2], we prove that the boundary value problems (1.1) and (1.5) or (1.6) has at least *three* positive solutions. If $i = 1$ and $a_1 = b_1 = 1$, then our results can improve and generalize the results of Li and Shen [8] in the case $\mathbb{T} = \mathbb{R}$. As an application, an example is given to illustrate the result.

Throughout this paper, it is assumed that

(H1) $f : \mathbb{R} \to \mathbb{R}^+$ is continuous, and does not vanish identically on any closed subinterval of $[0, T]_{\mathbb{T}}$, where $\mathbb{R}^+$ denotes the nonnegative real numbers;

(H2) $a_1 : \mathbb{T} \to \mathbb{R}^+$ is left dense continuous (i.e., $a_1 \in C_{ld}(\mathbb{T}, \mathbb{R}^+)$), and does not vanish identically on any closed subinterval of $[0, T]_{\mathbb{T}}$, where $C_{ld}(\mathbb{T}, \mathbb{R}^+)$ denotes the set of all left dense continuous functionals from $\mathbb{T}$ to $\mathbb{R}^+$.

For convenience, we list the following well-known definitions which can be found in [3,4].

Definition 1.1. For $t < \sup \mathbb{T}$ and $r > \inf \mathbb{T}$, define the forward jump operator σ and the back jump operator ρ, respectively,

$$\sigma(t) = \inf\{\tau \in \mathbb{T} | \tau > t\} \in \mathbb{T}, \qquad \rho(r) = \sup\{\tau \in \mathbb{T} | \tau < r\} \in \mathbb{T} \quad \text{for all } t, \ r \in \mathbb{T}.$$