Available online at www.sciencedirect.com

ScienceDirect

Nonlinear Analysis 69 (2008) 3931–3951

Nonlinear Analysis

www.elsevier.com/locate/na

Asymptotic patterns of a structured population diffusing in a two-dimensional strip[☆]

Peixuan Weng[a,*], Dong Liang[b], Jianhong Wu[b]

[a] School of Mathematics, South China Normal University, Guangzhou 510631, PR China
[b] Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada

Received 3 March 2007; accepted 18 October 2007

Abstract

In this paper, we derive a population model for the growth of a single species on a two-dimensional strip with Neumann and Robin boundary conditions. We show that the dynamics of the mature population is governed by a reaction–diffusion equation with delayed global interaction. Using the theory of asymptotic speed of spread and monotone traveling waves for monotone semiflows, we obtain the spreading speed c^*, the non-existence of traveling waves with wave speed $0 < c < c^*$, and the existence of monotone traveling waves connecting the two equilibria for $c \geq c^*$.

MSC: 34K30; 35K57; 35Q80; 92D25

Keywords: Structured population model; Monotone semiflows; Asymptotic patterns; Spreading speed; Traveling waves; Two-dimensional strip

1. Introduction

Aiello and Freedman [1] proposed the following system for a single species population with age stages:

$$w'(t) = \alpha e^{-\gamma r} w(t-r) - \beta w^2(t), \tag{1.1}$$

where α, β, γ and r are positive constants, w denotes the numbers of adult members of the population, and r is the time taken from birth to maturity. The first term of (1.1) represents the rate of recruitment into the adult population, and the second term represents the mortality rates of adult individuals. This system provides an alternative, and more realistic model for a single species than the logistic equation $w' = w(1-w)$. As shown in [1], all solutions of (1.1), other than the trivial solution, converge to the positive equilibrium solution w^*.

☆ Research partially supported by NSF of China and NSF of Guangdong Province (for PW), by NSERC of Canada (for DL and JW), by the Canada Research Chairs Program and by MITACS (for JW).
* Corresponding address: School of Mathematical Sciences, South China Normal University, Guangzhou 510631, Guangdong, PR China. Tel.: +86 20 85213533.
E-mail addresses: wengpx@scnu.edu.cn (P. Weng), dliang@mathstat.yorku.ca (D. Liang), wujh@mathstat.yorku.ca (J. Wu).

0362-546X/$ - see front matter
doi:10.1016/j.na.2007.10.027

By considering the diffusion in continuous space which is Fickian diffusion, (1.1) was generalized by AL-Omari and Gourley [2] to the following form

$$\frac{\partial w(x,t)}{\partial t} = D_m \Delta w(x,t) + \int_0^r \int_\Omega G(x,y,s) f(s) \mathrm{e}^{-\gamma s} b(w(y,t-s)) \mathrm{d}y \mathrm{d}s - d(w(x,t)) \tag{1.2}$$

subject to a Neumman boundary value condition on $\partial\Omega$, where D_m is a diffusion coefficient, d is the death function, Ω is a bounded domain, f is a probability function satisfying $\int_0^r f(s)\mathrm{d}s = 1$, G is a kernel yielded from solving the heat equation, and satisfies $\int_\Omega G(x,y,t)\mathrm{d}x = \int_\Omega G(x,y,t)\mathrm{d}y = 1$. In [2], AL-Omari and Gourley showed also the global attractivity of the positive steady state $\hat{w}$ of (1.2).

In the present article, we shall consider a similar system as in [2], which represents the population growth of a single species with age stages in a two-dimensional strip domain. In Section 2, a reaction–diffusion equation with delayed global interaction is derived for the mature population:

$$\begin{aligned} \frac{\partial w}{\partial t} &= D_m \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] - d_m w \\ &\quad + \mu \int_{\mathbb{R}} \int_0^L \Gamma(\alpha, x, z_x, y, z_y) b(w(t-r, z_x, z_y)) \mathrm{d}z_x \, \mathrm{d}z_y, \quad t > 0, (x,y) \in (0,L) \times \mathbb{R}, \\ Bw(t,x,y) &= 0, \quad t \ge 0, x = 0, L, \ y \in \mathbb{R}, \end{aligned} \tag{1.3}$$

where $b(\cdot)$ is the birth function, $Bw(t,x,y) = p(x)w(t,x,y) + \frac{\partial}{\partial n} w(t,x,y)$ is the boundary value condition including the Neumann ($p(0) = p(L) = 0$) and Robin ($p(0) \ge 0$, $p(L) \ge 0$, $[p(0)]^2 + [p(L)]^2 \ne 0$) boundary value conditions respectively. Although we only consider the case when the death function $d(w) = d_m w$, we change Ω into a unbounded strip domain $(0, L) \times \mathbb{R}$ which makes us able to discuss two very important asymptotic properties (traveling waves and spread speed) of the population system (1.3) as $t = \infty$. The boundary value conditions here are also more abundant than that in [2].

In population dynamics, two key elements to the developmental process seems to be the appearance of a traveling wave and the spread speed (or, asymptotic speed of spread). A traveling wave is a special solution which travels without any change in shape. Traveling wave solutions have been widely studied for reaction–diffusion equations [18,22,28], integral and integro-differential equations [4–6], lattice systems [25,27]. The concept of spreading speed was first introduced by Aronson and Weinberger [3] for reaction–diffusion equations, and also applied to integro-differential equations, integral equations, lattice systems and systems of recursions. See [6,11,13,16,20,21,23,24,26] and the references therein. The spreading speed is a threshold constant $c^* > 0$ which gives an important description of the long time behaviors of the population systems either for $c \in (0, c^*)$ or $c \in (c^*, \infty)$. Taking (1.3) as an example, the spreading speed c^* is a number in the sense that $\lim_{t\to\infty, |y| \ge ct} w(t,x,y) = 0$ uniformly on $x \in [0, L]$ if $c > c^*$ and the initial function is zero for y outside a bounded interval, and that $\lim_{t\to\infty, |y|\le ct} w(t,x,y) = w^+(x)$ ($w^+(x)$ is the positive equilibrium of (1.3)) uniformly on $x \in [0, L]$ if $c \in (0, c^*)$ and the initial function is not identical to zero (see Theorem 3.2 in this article).

Recently, the theory of asymptotic speeds of spread and monotone traveling waves for monotone semiflows (discrete or continuous time) has been developed by Liang and Zhao [12] in such a way that it can be applied to various evolution equations admitting the comparison principle. For every population dynamical system admitting the comparison principle, if the solution of the initial problem exists and is unique, then all solutions form a monotone semiflow $\{Q_t\}_{t=0}^\infty$ which has an asymptotic speed of spread $c^* > 0$ under some conditions (A1)–(A5). Furthermore an estimates of c^* can be given by the linearized approach. On the other hand, the existence of traveling waves above c^* and their non-existence below c^* can be obtained with an extra condition (A6), and thus c^* is also the minimal wave speed of the system.

In this paper, we shall apply the theory mentioned above to the population model (1.3) to obtain the asymptotic speed and monotone traveling waves by imposing a sublinear condition to the birth function b. The application of this theory seems very technical and tricky. It is organized as follows. In Section 2, we give a derivation of model (1.3) and discuss the dynamical structure of the steady states. We show the existence and the global attractivity of the positive steady state by using the theory of functional differential equations in abstract space. The main results are presented in Section 3. We first investigate the existence of solutions for (1.3) and show that the system (1.3) satisfies