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Periodic solutions to a second order p-Laplacian neutral functional differential system☆

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Abstract

By analyzing some properties of the linear difference operator $A : [Ax](t) = x(t) - Cx(t-\tau)$ first, and then using an extension of Mawhin's continuation theorem, a second order p-Laplacian neutral functional differential system as follows

$$\frac{d}{dt}\phi_p[(x(t) - Cx(t-\tau))'] = f(t, x(t), x(t-\mu(t)), x'(t))$$

is studied. Some new results on the existence of periodic solutions is obtained. The result is related to the deviating arguments τ and μ. Meanwhile, the approaches to *estimate a priori bounds* of periodic solutions are different from the corresponding ones of the known literature.

MR Subject Classification: 34B15; 34B13

Keywords: Periodic solution; Coincidence degree; p-Laplacian differential equation; Deviating argument

1. Introduction

The aim of this paper is to consider a second order p-Laplacian neutral functional differential system as follows

$$\frac{d}{dt}\phi_p[(x(t) - Cx(t-\tau))'] = f(t, x(t), x(t-\mu(t)), x'(t)), \tag{1.1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^\top$, $\varphi_p(x) = |x|^{p-2}x = \left(\sqrt{\sum_{i=1}^n x_i^2}\right)^{p-2} x$, $p > 1$, $\tau \in R$ are constants, $f : R^4 \to R^n$, $\mu : R \to R$ are continuous with $f(t+T, x_0, x_1, x_2) \equiv f(t, x_0, x_1, x_2)$, $\forall (x_0, x_1, x_2) \in R^3$ and $\mu(t+T) \equiv \mu(t)$, $C = [c_{ij}]_{n\times n}$ is a real constant matrix.

In the past few years, there were plenty of results on the existence of periodic solutions to delay Duffing or Liénard or Rayleigh type equations, (see [1–4] and the references therein). Recently, the problems of periodic solutions to some kinds of p-Laplacian delay differential equations have attracted much attention from researchers [5,6]. For example,

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Cheung and Ren studied the existence of T-periodic solutions to a one dimension p-Laplacian Liénard equation with a deviating argument in [5] as follows

$$(\varphi_p(x'(t)))' + f(x(t))x'(t) + g(x(t-\tau(t))) = e(t).$$

Since $(\varphi_p(x'(t)))'$ is no longer linear, Mawhin's continuation theorem [7] does not be apply directly. In order to get around this difficulty, Cheung and Ren in [5] designed a new technique of tackling the problem, namely, to translate the p-Laplacian equation into a two-dimensional system as follows

$$\begin{cases} x_1'(t) = \varphi_q(x_2(t)) = |x_2(t)|^{q-2}x_2(t) \\ x_2'(t) = -f(x_1(t))\varphi_q(x_2(t)) - g(x_1(t-\tau(t))) + e(t), \end{cases}$$

where $q > 1$ is a constant with $1/p + 1/q = 1$, for which Mawhin's continuation theorem can be applied. As the neutral differential equation frequently comes into play in many practical situations(for example, it is used to describe electrical mechanical phenomena of networks containing lossless transmission lines), it is natural to try to consider the existence of periodic solutions to some kinds neutral differential equations. In [8–12], the authors studied the problem of periodic solutions for neutral functional differential equation (NFDE) as follows

$$\frac{\mathrm{d}}{\mathrm{d}t} Dx_t = g(x_t), \tag{1.2}$$

where $x_t(\theta) = x(t+\theta), \forall\theta \in [-\tau, 0]$, $D : C([-\tau, 0], R^n) \to R^n$ is a linear difference operator. However, the crucial condition that D is stable, i.e., the zero solution of the functional equation

$$Dy_t = 0, \qquad y_0 = \phi \in \{C([-\tau, 0], R^n) : D\varphi = 0\}$$

is uniformly asymptotically stable, was needed in those papers. Very recently, by using Mawhin's continuation theorem, Lu and Ge in [13,14] studied the following neutral differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}(u(t) - ku(t-\tau)) = g_1(u(t)) + g_2(u(t-\tau_1)) + p(t)$$

and

$$(u(t) - ku(t-\tau))'' + f(u(t))u'(t) + \sum_{j=1}^{n} \beta_j(t)g(u(t-\gamma_j(t))) = p(t).$$

Under the condition of constants $|k| \neq 1$, the authors obtained some new results on the existence of periodic solutions. But as far as we know, the problem of periodic solution for p-Laplacian neutral functional differential equations (1.1) has not been studied until now. The obvious difficulty lies in the following two respects. The first is that since the leading term contains a p-Laplacian difference operator, the properties of the solution is much more complex than the corresponding ones of delay equation; the second is that *a priori bounds* of periodic solutions is not easy to estimate. In order to overcome these difficulties, we analyze some properties of the linear difference operator $A : [Ax](t) = x(t) - Cx(t-\tau)$ first, which will be used to estimate *a priori* bounds of periodic solutions in Section 3. The second is that we use a new theorem [15], named an extension of Mawhin's continuation theorem, to investigate the existence of periodic solutions to Eq. (1.1), which is different from the methods used in [5,6]. The significance of the present paper is that the result (Theorem 3.2) is related to the deviating arguments τ and $\mu(t)$. Furthermore, we do not necessarily need that the linear difference operator is stable, which is different from the corresponding ones of [9–12].

2. Main lemmas

In order to use the extension of Mawhin's continuation theorem [15], we first recall it.

Let X and Z be Banach spaces with norms $\|\cdot\|_X$ and $\|\cdot\|_Y$, respectively. A continuous operator $M : X \cap \operatorname{dom} M \to Z$ is said to be *quasi-linear* if

(a)

$$\operatorname{Im} M := M(X \cap \operatorname{dom} M) \tag{2.1}$$

is a closed subset of Z;