



Eventual stability properties in a non-autonomous model of population dynamics

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ABSTRACT

We prove that $(\lambda^*, C/\lambda^*)$ is an eventually uniform-asymptotically stable point in the large of the system

$$\begin{aligned}\dot{L} &= C - LG, \\ \dot{G} &= (L - \lambda(t))G.\end{aligned}$$

on the quadrant $\{(L, G) : L \geq 0, G > 0\}$. Here function $\lambda(t)$ is positive and $\lambda(t) \rightarrow \lambda^* > 0$ as $t \rightarrow \infty$. The study was inspired by observations of distributions of peculiar carnivore and herbivore fish species in Lake Tanganyika.

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1. The model

This study is inspired by observations of distributions of peculiar fish species in Lake Tanganyika [1]. In this lake, extraordinarily in the world, the carnivore fishes have asymmetric faces. Actually they can be divided into two groups: those with mouths turned to left and right, respectively. The members of the first group (with “left mouth”) attack their prey mainly from the left while the other group prefers to attack from the right. One has observed that the prey, actually herbivore fish, try to adapt to the distribution of left and right attacks against them. Their strategy seems to be rather rigid: a given individual herbivore does not change his preference of paying more attention to against attacks from the left or from the right during his life.

The above situation suggests several mathematical models concerning the development of population numbers or total weights of the groups of left or right mouth carnivores and the groups of herbivores with various strategies against them.

Throughout this work let \mathcal{I} and \mathcal{K} be two (finite) index sets representing the groups of herbivores and carnivores, respectively. In the case described above we simply have two elements in both \mathcal{I} and \mathcal{K} ; namely, the groups of herbivores with right and left attention preference in \mathcal{I} , and the groups of carnivores with right and left distorted mouths in \mathcal{K} . However, for technical reasons we may not restrict the numbers of the subspecies. In the sequel we shall write $n_i = n_i(t)$ for the number of the herbivores at time t in group $i \in \mathcal{I}$. Similarly, $m_k = m_k(t)$ will denote the number of the carnivores at time t in group $k \in \mathcal{K}$. Our basic idea for constructing differential equations for these function is the following.

The whole system of the nutrition chain consisting of plants, herbivores and carnivores is supported by the energy flow provided by the Sun. We assume that the intensity of this flow is constant, and furthermore we assume that the growth

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of the total mass of the plants due to the constant solar energy flow is C per time unit. Plants will be eaten by herbivores: we assume that an individual with weight w consumes the percentage $\alpha(w)$ from the total mass of plants during a time unit. Here we make a crucial additional assumption: each group $i \in \mathcal{I}$ consists of individuals with the same weight $w_i(t)$ at the time t . Similarly we assume that each carnivore group $k \in \mathcal{K}$ consists of individuals with weight $u_k = u_k(t)$. By writing $K = K(t)$ for the total mass of plants at time t , our hypothesis concerning plants and herbivores can be formulated as follows:

$$\dot{K} = C - \sum_i n_i \alpha(w_i) K.$$

In this note we are only concerned with one period (actually one year) development without mating. We assume that carnivores do not die during this time, thus their numbers $m_k(t)$ are constant in time. On the other hand, the number of herbivores will be decreased by the carnivores. We assume that the various groups are homogeneously located in the lake and the number of attacks is proportional to their density. That is, with some constant ρ , in a time unit we have $\rho n_i m_k$ attacks by carnivores of type k against herbivores of type i . Concerning the outcome of such an attack, we assume that a herbivore from the group i with weight w will be eaten by a carnivore from the group k of weight u with a probability $\beta^{(i,k)}(w, u)$. Thus

$$\dot{n}_i = - \sum_k \rho \beta^{(i,k)}(w_i, u_k) n_i m_k.$$

Let $\gamma(e, w)$ denote the weight that a herbivore of weight w gains by eating e amount of plants. The weight that a carnivore loses without eating during a time unit is denoted by $\tilde{\gamma}(w)$. Thus

$$\dot{w}_i = \gamma(\alpha(w_i)K, w_i) - \tilde{\gamma}(w_i).$$

As we have assumed that during the period of development the carnivores do not die (they just lose weight), we have:

$$\dot{m}_k = 0.$$

Let $\delta(e, u)$ denote the weight that a carnivore of weight u gains by eating e amount of herbivores. The weight that a carnivore loses without eating during a time unit is denoted by $\tilde{\delta}(u)$. Thus

$$\dot{u}_k = \delta \left(\sum_i \rho \beta^{(i,k)}(w_i, u_k) w_i n_i m_k, u_k \right) - \tilde{\delta}(u_k).$$

2. Simplification of the model

We assume that some functions in the model are linear: $\alpha(w) = \alpha w$, $\gamma(e, w) = \gamma e$, $\delta(e, u) = \delta e$, $\beta^{(i,k)}(w, u) \equiv \beta^{(i,k)}$, $\tilde{\gamma}(w) = \tilde{\gamma} w$ and $\tilde{\delta}(u) = \tilde{\delta} u$. Under these assumptions we have the equations

$$\dot{K} = C - \sum_i n_i \alpha w_i K,$$

$$\dot{n}_i = - \sum_k \beta^{(i,k)} n_i m_k \rho,$$

$$\dot{w}_i = \gamma \alpha w_i K - \tilde{\gamma} w_i,$$

$$\dot{u}_k = \delta \rho \sum_i \beta^{(i,k)} w_i n_i m_k - \tilde{\delta} u_k.$$

We introduce the new variables $x_i := n_i w_i$ and $y_k := m_k u_k$ for the total weight of the herbivores and the carnivores, respectively. For the new variables we have

$$\begin{aligned} \dot{x}_i &= \dot{n}_i w_i + n_i \dot{w}_i \\ &= - \sum_k \beta^{(i,k)} n_i m_k \rho w_i + n_i \gamma \alpha w_i K - n_i \tilde{\gamma} w_i \\ &= \left[\alpha \gamma K - \tilde{\gamma} - \rho \sum_k \beta^{(i,k)} m_k \right] x_i \end{aligned}$$

and

$$\begin{aligned} \dot{y}_k &= m_k \dot{u}_k \\ &= m_k^2 \delta \rho \sum_i \beta^{(i,k)} w_i n_i - m_k \tilde{\delta} u_k \\ &= m_k^2 \delta \rho \sum_i \beta^{(i,k)} x_i - \tilde{\delta} y_k. \end{aligned}$$

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