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Existence and stability of almost periodic solutions of Hopfield neural networks with continuously distributed delays

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ABSTRACT

In this paper, the global stability and almost periodicity are investigated for Hopfield neural networks with continuously distributed neutral delays. Some sufficient conditions are obtained for the existence and globally exponential stability of almost periodic solution by employing fixed point theorem and differential inequality techniques. The results of this paper are new and they complement the previously known ones. Finally, an example is given to demonstrate the effectiveness of our results.

1. Introduction and preliminaries

It is well known that Hopfield neural networks ([1,2]) have been paid great attention in the past decade due to the potential applications in many fields such as signal processing, image processing, pattern recognition and optimization, etc. So many authors in recent years have great interest in the dynamics and applications of Hopfield neural networks. In particular, there have been extensive results on the problem of the existence and stability of periodic and almost periodic solutions of Hopfield neural networks in the literature. We refer the readers to [3–12] and the references cited therein.

Due to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of the neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions. Therefore, it is important and, in effect, necessary to introduce a new type of network — neural networks of neutral type. Such networks arise in high speed computers where nearly lossless transmission lines are used to interconnect switching circuits. Also, the neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar. Recently, the study of the neural networks with neutral delays has received much attention, see, for instance, Refs., [13–16] and the references cited therein.

Recently, Liu [17] studied the following Hopfield neural networks with continuously distributed delays

$$x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} b_{ij}(t) \int_0^{\infty} K_{ij}(u)g_j(x_j(t-u))\mathrm{d}u + I_i(t), \quad i = 1, 2, \ldots, n, \tag{1.1}$$

and obtained sufficient conditions for the existence and exponential stability of the almost periodic solutions of system (1.1).

In this paper, we consider the following models for Hopfield neural networks with continuously distributed neutral delays

$$x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) \int_0^{\infty} K_{ij}(u)f_j(x_j(t-u))\mathrm{d}u + \sum_{j=1}^{n} b_{ij}(t) \int_0^{\infty} D_{ij}(u)g_j(x_j'(t-u))\mathrm{d}u + I_i(t), \tag{1.2}$$

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where $i = 1, 2, \ldots, n$, $x_i(t)$ denotes the potential (or voltage) of cell i at time t; $c_i(t) > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t; $a_{ij}(t)$ and $b_{ij}(t)$ represent the delayed strengths of connectivity and neutral delayed strengths of connectivity between cell i and j at time t, respectively; f_j and g_j are the activation functions in system (1.2); $I_i(t)$ is an external input on the ith unit at time t.

To the best of our knowledge, no paper in the literature has investigated the globally exponential stability and existence of almost periodic solution for system (1.2). Hence, our goal in this paper is to study the almost periodic solution of Hopfield model (1.2). By applying fixed point theorem and differential inequality techniques, we give some sufficient conditions ensuring the existence and globally exponential stability of continuously differentiable almost periodic solution of system (1.2), which are new and complement the previously known results.

We list some assumptions which will be used in this paper.

(H_1) $c_i(t) > 0$, $a_{ij}(t)$, $b_{ij}(t)$, and $I_i(t)$ are almost periodic functions, $i, j = 1, 2, \ldots, n$.

(H_2) There exist positive constants $L_j > 0$, $l_j > 0$ such that

$$|f_i(x) - f_i(y)| \le L_i|x - y|, \qquad |g_i(x) - g_i(y)| \le l_i|x - y|, \quad \text{for all } x, y \in \mathbb{R},\ i = 1, 2, \ldots, n.$$

(H_3) For $i, j \in \{1, 2, \ldots, n\}$, the delay kernels $K_{ij}, D_{ij} : [0, \infty) \to R$ are continuous, integrable and satisfy

$$\int_0^\infty |K_{ij}(u)|\mathrm{d}u \le \overline{K}_{ij}, \qquad \int_0^\infty |D_{ij}(u)|\mathrm{d}u \le \overline{D}_{ij}.$$

(H_4) There exists a constant $\lambda_0 > 0$ such that

$$\int_0^\infty |K_{ij}(u)|\mathrm{e}^{\lambda_0 u}\mathrm{d}u < +\infty, \qquad \int_0^\infty |D_{ij}(u)|\mathrm{e}^{\lambda_0 u}\mathrm{d}u < +\infty, \quad i, j \in \{1, 2, \ldots, n\}.$$

The initial conditions associated with (1.2) are of the form

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0],\ i = 1, 2, \ldots, n, \tag{1.3}$$

where $\varphi_i(\cdot)$ denotes the real-valued bounded continuously differentiable function defined on $(-\infty, 0]$.

Definition 1.1 ([18,19]). Let $u : R \to R^n$ be continuous in t. $u(t)$ is said to be almost periodic on R if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{\delta : |u(t + \delta) - u(t)| < \varepsilon,\ \forall t \in R\}$ is relatively dense, i.e., for $\forall \varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $|u(t+\delta)-u(t)| < \varepsilon$, for $\forall t \in R$.

Definition 1.2 ([18,19]). If $u(t) : R \to R^n$ is continuously differentiable in t, $u(t)$ and $u'(t)$ are almost periodic on R, then $u(t)$ is said to be a continuously differentiable almost periodic function.

Definition 1.3 ([18,19]). Let $x \in R^n$ and $Q(t)$ be a $n \times n$ continuous matrix defined on R. The linear system

$$x'(t) = Q(t)x(t) \tag{1.4}$$

is said to admit an exponential dichotomy on R if there exist positive constants k, α, projection P and the fundamental solution matrix $X(t)$ of (1.4) satisfying

$$\|X(t)PX^{-1}(s)\| \le k\mathrm{e}^{-\alpha(t-s)}, \quad \text{for } t \ge s,$$
$$\|X(t)(I - P)X^{-1}(s)\| \le k\mathrm{e}^{-\alpha(s-t)}, \quad \text{for } t \le s.$$

Definition 1.4. Let $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^{\mathrm{T}}$ be a continuously differentiable almost periodic solution of system (1.2) with initial value $\varphi^* = (\varphi_1^*(t), \varphi_2^*(t), \ldots, \varphi_n^*(t))^{\mathrm{T}}$. If there exist constants $\lambda > 0$ and $M \ge 1$ such that for every solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}}$ of system (1.2) with any initial value $\varphi = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^{\mathrm{T}}$,

$$\begin{aligned}\|x(t) - x^*(t)\|_1 &= \max\{\|x(t) - x^*(t)\|, \|x'(t) - {x^*}'(t)\|\}\\ &\le M\|\varphi - \varphi^*\|_1\mathrm{e}^{-\lambda t} = M\max\{\|\varphi - \varphi^*\|, \|\varphi' - {\varphi^*}'\|\}\mathrm{e}^{-\lambda t}, \quad \forall t > 0,\end{aligned}$$

where $\|x(t) - x^*(t)\| = \max_{1\le i\le n} |x_i(t) - x_i^*(t)|$, and $\|\varphi - \varphi^*\| = \sup_{-\infty\le s\le 0} \max_{1\le i\le n} |\varphi_i(s) - \varphi_i^*(s)|$. Then $x^*(t)$ is said to be globally exponential stable.

Lemma 1.1 ([18,19]). *If the linear system (1.4) admits an exponential dichotomy, then almost periodic system*

$$x'(t) = Q(t)x(t) + g(t) \tag{1.5}$$

has a unique almost periodic solution $x(t)$, and

$$x(t) = \int_{-\infty}^t X(t)PX^{-1}(s)g(s)\mathrm{d}s - \int_t^{+\infty} X(t)(I - P)X^{-1}(s)g(s)\mathrm{d}s. \tag{1.6}$$