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Global asymptotic stability analysis of discrete-time Cohen–Grossberg neural networks based on interval systems☆

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Abstract

The global asymptotic stability of discrete-time Cohen–Grossberg neural networks (CGNNs) with or without time delays is studied in this paper. The CGNNs are transformed into discrete-time interval systems, and several sufficient conditions for asymptotic stability for these interval systems are derived by constructing some suitable Lyapunov functionals. The conditions obtained are given in the form of linear matrix inequalities that can be checked numerically and very efficiently by using the MATLAB LMI Control Toolbox. Finally, some illustrative numerical examples are provided to demonstrate the effectiveness of the results obtained.

Keywords: Cohen–Grossberg neural network; Global asymptotic stability; Time-delay system; Interval system; Discrete time; Linear matrix inequality (LMI)

1. Introduction

Cohen–Grossberg neural networks (CGNNs) were first introduced by Cohen and Grossberg [1] in 1983. This class of networks has been the subject of extensive investigation because of their many important applications, such as in pattern recognition, associative memory and combinatorial optimization. Such applications depend heavily on the dynamical behaviors. Thus, the analysis of the dynamical behaviors, such as stability, is a necessary step for practical design of neural networks. Recently, many scientific and technical workers have been joining in the studies with great interest, and various interesting results for CGNNs with delays and without delays have been reported [2–6]. In general, the continuous-time CGNN model is described by the set of ordinary differential equations [1]

$$\dot{x}_i(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} c_{ij} f_j(x_j(t)) + J_i\right], \quad i = 1, 2, \ldots, n, \tag{1}$$

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where $x_i(t)$ is the state variable of the ith neuron, $a_i(\cdot)$ represents an amplification function, $b_i(\cdot)$ is a well behaved function, $(c_{ij})_{n\times n}$ denotes the connection matrix in which c_{ij} represents the strength of connection from neuron i to j, $f_j(\cdot)$ is called an activation function indicating how the jth neuron responds to its input, and J_i denotes the ith component of an external input source introduced from outside the network to the cell i. When the delays are introduced in CGNN (1), we obtain the following system [6]:

$$\dot{x}_i(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} c_{ij} f_j(x_j(t-\eta_{ij}(t))) + J_i\right], \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\eta_{ij}(t)$ is the transmission delay. Xiong et al. [2] formulated the following discrete-time versions of the system (1) and (2):

$$x_i(k+1) = x_i(k) - a_i(x_i(k))\left[b_i(x_i(k)) - \sum_{j=1}^{n} c_{ij} f_j(x_j(k)) + J_i\right], \quad i = 1, 2, \ldots, n, \tag{3}$$

and

$$x_i(k+1) = x_i(k) - a_i(x_i(k))\left[b_i(x_i(k)) - \sum_{j=1}^{n} c_{ij} f_j(x_j(k-\eta_{ij}(k))) + J_i\right] \quad i = 1, 2, \ldots, n, \tag{4}$$

where $\eta_{ij}(k)$ represents the time delay and is a positive integer with $\eta_{ij}(k) \le h$. The initial conditions associated with Eq. (3) are of the form

$$x_i(k) = x_i(0), \quad i = 1, 2, \ldots, n, \tag{5}$$

and the initial conditions associated with Eq. (4) are of the form

$$x_i(k) = \varpi(k), \quad \forall k \in [-h, 0], \tag{6}$$

where $\varpi_i(k)$ is the given discrete-time function on $[-h, 0]$.

For system (3) and (4), we make the following assumptions:

Assumption A. Suppose that $a_i(\cdot)$, $b_i(\cdot)$ and $f_j(\cdot)$ are Lipschitz continuous; furthermore, $0 < \underline{a}_i \le a_i(x_i(k)) \le \overline{a}_i$, $0 < \gamma_i \le b_i(x_i(k))/x_i(k) < +\infty$, and $0 \le f_j(x_j(k))/x_j(k) \le \sigma_j$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$.

It is worth noting that although Ref. [2] has investigated the global exponential stability of CGNN (3) and (4), the asymptotic stability conditions are also required widely in engineering fields. In this paper, our main purpose is to derive some criteria for global asymptotic stability for the discrete-time CGNN (3) and (4) by a new method. We first transform the CGNNs into interval systems, and analyze their stability on the basis of the linear matrix inequality (LMI) approaches. The global asymptotic stability of the discrete-time CGNNs is judged by solving some LMIs by using MATLAB LMI Control Toolbox.

2. Main results

Throughout this paper, $\boldsymbol{I}$ denotes the identity matrix of appropriate order, $*$ denotes the symmetric parts. If $\boldsymbol{\Lambda}$ is a diagonal positive (or semi-positive) definite matrix, $\boldsymbol{\Lambda}^{\frac{1}{2}}$ denotes a diagonal positive (or semi-positive) definite matrix of which the diagonal element is a square root of $\boldsymbol{\Lambda}$'s. The notation $\boldsymbol{X} > \boldsymbol{Y}$ and $\boldsymbol{X} \ge \boldsymbol{Y}$, where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are matrices of the same dimensions, means that the matrix $\boldsymbol{X} - \boldsymbol{Y}$ is positive definite and semi-positive definite, respectively.

From Theorem 2.1 in [2], system (3) (or system (4)) always has an equilibrium point under Assumption A. Suppose $\boldsymbol{x}^* = (x_1^*(k), x_2^*(k), \ldots, x_n^*(k))^{\mathrm{T}}$ to be an equilibrium point of system (3) (or (4)), let $y_i(k) = x_i(k) - x_i^*(i = 1, 2, \ldots, n)$; then we can rewrite Eqs. (3) and (4) as

$$y_i(k+1) = y_i(k) - \alpha_i(y_i(k))\left[\beta_i(y_i(k)) - \sum_{j=1}^{n} c_{ij} g_j(y_j(k))\right] \tag{7}$$