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Persistence and extinction of disease in non-autonomous SIRS epidemic models with disease-induced mortality ☆

Zhidong Teng*, Yanping Liu, Long Zhang

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, People's Republic of China

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Abstract

In this paper, non-autonomous SIRS epidemic models with bilinear incidence and disease-induced mortality are studied. Under the quite weak assumptions, the sufficient and necessary conditions on the permanence and strong persistence of the disease and the sufficient condition on the extinction of the disease are established. Some new threshold values of the integral form R_0^*, R_1^* and R_2^* are obtained. We prove that the disease is permanent if and only if $R_0^* > 0$, and if $R_1^* \le 0$ or $R_2^* < 0$, then the disease is extinct. As applications of the main results, we discuss the periodic and almost periodic models. The corresponding basic reproductive numbers R_0 are obtained. We show that if $R_0 > 1$, then the disease is permanent and if $R_0 \le 1$, then the disease is extinct.

MSC: 34D25; 92D20

Keywords: Non-autonomous SIRS model; Disease; Threshold value; Persistence; Permanence; Extinction

1. Introduction

In recent years, the autonomous epidemic models have been investigated extensively. The basic and important research subjects for these models are the calculation of the reproductive number of disease or threshold values, the local and global stability of disease-free equilibrium and endemic equilibrium, the existence, uniqueness and stability of periodic oscillation of disease, Hopf bifurcation, the persistence, permanence and extinction of disease, etc. Many important results with regard to these subjects can be found in many articles, for example, see [1,2,4,6,7] and the references cited therein. Particularly, the works in allusion to various types of SIRS epidemic models can be found in [3,5,8–11,13,14,17–20,25,26] and the references cited therein.

However, as we know well, the non-autonomous phenomenon is so prevalent and all-pervasive in the real life that modelling biological proceeding under non-autonomous environment should be more realistic than autonomous situation. Particularly, it will be more identical with the sound background as we consider long-term dynamical behavior of an epidemic system, the parameters of the system usually will change with time. Therefore, the research

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* Corresponding author.
E-mail address: zhidong@xju.edu.cn (Z. Teng).

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on the non-autonomous epidemic models also plays very important role in epidemiology. We see that there have been some research works on the non-autonomous epidemic models (see [12,15,21,22,24]).

In [12], the authors studied the following non-autonomous SEIRS epidemic model with nonlinear incidence rate

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= \mu(t) - \lambda(t)I^pS^q - \mu(t)S + \delta(t)R,\\ \frac{\mathrm{d}E}{\mathrm{d}t} &= \lambda(t)I^pS^q - \mu(t)I - \varepsilon(t)E,\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \varepsilon(t)E - \mu(t)I - \gamma(t)I,\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma(t)I - \mu(t)R - \delta(t)R,\end{aligned} \tag{1}$$

where the letters S, E, I and R stand, respectively, for susceptible, exposed, infectious and recovered. Functions $\mu(t)$, $\lambda(t)$, $\varepsilon(t)$, $\gamma(t)$ and $\delta(t)$ are nonnegative continuous for all $t \geq 0$, and p, q are two positive constants. Applying the positivity of certain vector-value function, the authors established the sufficient conditions on the extinction of the disease for model (1) (see Theorems 3 and 4 in [12]).

In [15], the authors studied the following very general non-autonomous SIR epidemic model with nonlinear incidence and disease-induced mortality

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= Nf(t,N) - g(t,N)S - \phi(t,S,I,N),\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \phi(t,S,I,N) - [g(t,N) + \gamma(t,N) + \alpha(t,N)]I,\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma(t,N)I - g(t,N)R,\\ N &= S + I + R.\end{aligned}$$

By using the linearization method, a threshold value which determines the disease is weakly persistent or extinct is established (see Theorem 2.4.2 in [15]).

In [21,22], the authors studied the persistence and extinction of the disease for the following non-autonomous SIRS epidemic model with bilinear incidence

$$\begin{aligned}N &= S + I + R\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \alpha(t)SI - \mu(t)I - \gamma(t)I\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma(t)I - \mu(t)R - \xi(t)R,\end{aligned} \tag{2}$$

where $N = N(t)$ is a known continuous, bounded and nonnegative function defined on $R_+ = [0, \infty)$, denotes the size of the total population at time t. N is divided into three compartments: susceptible S, infective I and recovered R. Functions $\alpha(t)$, $\mu(t)$, $\gamma(t)$ and $\xi(t)$ are nonnegative continuous and bounded on R_+. Applying the theory of the persistence and permanence for non-autonomous semiflows in the population biology which is developed in [21,22], the author established the sufficient conditions of the persistence and extinction of the disease for model (2) (see Theorems 3.3–3.5 in [21]).

In [24], motivated by the works [21,22], the authors continue to investigate model (2). By improving the research method given in [23], they established the sufficient and necessary conditions of the integral form on the permanence of the disease and the sufficient conditions on the extinction of the disease for model (2) (see Theorems 3.3–3.5 in [24]). We can easily see that these results are the very interesting improvement of the main results given in [21,22].