



Positive periodic solutions for a class of neutral delay Gause-type predator–prey system[☆]

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ABSTRACT

By using a continuation theorem based on coincidence degree theory, we establish easily verifiable criteria for the existence of positive periodic solutions for neutral delay Gause-type predator–prey system

$$\begin{cases} x'(t) = x(t)[r(t) - a(t)x(t - \sigma_1) - \rho x'(t - \sigma_2)] - \phi(t, x(t))y(t - \tau_1(t)), \\ y'(t) = y(t)[-d(t) + h(t, x(t - \tau_2(t)))]. \end{cases}$$

In addition, our results are applicable to neutral delay predator–prey systems with different types of functional responses such as Holling-type II and Ivlev-type.

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1. Introduction

Modelling of ecological systems has received a great deal of attention from theoretical ecologists in the last few decades. Much focus has been on mathematical models of these systems, since they have substantially contributed to the understanding of the dynamics of systems by forging strong links between models and available data. Mathematical analysis of such models frequently presents interesting and challenging aspects of dynamical systems theory.

There is a large body of literature on population dynamics in ecological modelling, particularly in predator–prey systems [1–22]. A well-known model of such systems is the predator–prey model of Gause-type with a monotonic-bounded functional response [5]. The dynamics of this model is described by the following differential equations:

$$\begin{cases} x'(t) = rx(t) \left(1 - \frac{x(t)}{K}\right) - y(t)\phi(x(t)), \\ y'(t) = y(t)(-D + \mu\phi(x(t))), \end{cases} \quad (1.1)$$

where x and y are the prey and the predator population size, respectively. The parameter r is the prey's intrinsic growth rate which describes the exponential growth of the prey population at low densities. The carrying capacity K represents the prey biomass at equilibrium in the absence of predators. The parameters μ and D are the conversion rate of prey to predator and predator death rate, respectively. The function ϕ is called the functional response of predator to prey, and describes the change in the rate of exploitation of prey by a predator as a result of a change in the prey density. Indeed, the growth of the predator is enhanced in the presence of the prey by an amount proportional to the number of prey. Thus, this functional

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response can be interpreted as the proportion of prey that is eaten by the predator. A more detailed biological description of this model can be found in [5,14].

Most of the published studies have considered the model (1.1) with functional responses which satisfy the following hypotheses [6,13,16]:

- (A₁) $\phi(0) = 0$;
- (A₂) $\phi'(x) > 0$, for $x \geq 0$;
- (A₃) $\phi''(x) < 0$, for $x \geq 0$;
- (A₄) $\lim_{x \rightarrow +\infty} \phi(x) = c < \infty$.

These studies have provided a detailed dynamics analysis of (1.1) with different types of functional responses such as sigmoidal, Holling-types II and III, and Ivlev-type. However, they neglect the effect of seasonality on the dynamics of the model.

In [12], the model (1.1) is extended to incorporate a seasonal functional response $\phi(t, x)$ as:

$$\begin{cases} x'(t) = rx(t) \left(1 - \frac{x(t)}{K} \right) - y(t)\phi(t, x(t)), \\ y'(t) = y(t)(-D + \mu\phi(t, x(t))), \end{cases} \tag{1.2}$$

where $\phi(t, x)$ satisfies the following conditions:

- (A₁^{*}) $\phi(t, 0) = 0$, for $t \in R$;
- (A₂^{*}) $\phi_x(t, x) > 0$, for $x > 0, t \in R$;
- (A₃^{*}) $\phi_{xx}(t, x) < 0$, for $x > 0, t \in R$;
- (A₄^{*}) $\lim_{x \rightarrow +\infty} \phi(t, x) = c < \infty$.

Under the assumptions (A₁^{*})–(A₄^{*}), the functional response ϕ covers many examples appearing in the literature [8,11].

Recently, Ding and Jiang [3] studied the existence of positive periodic solutions for the following delayed Gause-type predator–prey systems:

$$\begin{cases} x'(t) = x(t)f \left(t, \int_{-\gamma_0}^{-\gamma} x(t + \theta) d\mu(\theta) \right) - g(t, x(t))y(t - \tau(t)), \\ y'(t) = y(t)[-d(t) + h(t, x(t - \sigma(t)))]. \end{cases} \tag{1.3}$$

In 1991, Kuang [10] studied the local stability and oscillation of the following neutral delay Gause-type predator–prey system

$$\begin{cases} x'(t) = rx(t) \left[1 - \frac{x(t - \tau) + \rho x'(t - \tau)}{K} \right] - y(t)p(x(t)), \\ y'(t) = y(t)[- \alpha + \beta p(x(t - \sigma))]. \end{cases} \tag{1.4}$$

As pointed out by Freedman and Wu [23] and Kuang [9], it would be of interest to study the existence of periodic solutions for periodic systems with time delay. The periodic solutions play the same role played by the equilibria of autonomous systems. In addition, in view of the fact that many predator–prey systems display sustained fluctuations, it is thus desirable to construct predator–prey models capable of producing periodic solutions.

In this paper, motivated by the above work, we shall study the existence of positive periodic solutions of the following more general neutral delay Gause-type predator–prey system:

$$\begin{cases} x'(t) = x(t) [r(t) - a(t)x(t - \sigma_1) - \rho x'(t - \sigma_2)] - \phi(t, x(t))y(t - \tau_1(t)), \\ y'(t) = y(t)[-d(t) + h(t, x(t - \tau_2(t)))]. \end{cases} \tag{1.5}$$

For convenience, we will use the notations:

$$|f|_0 = \max_{t \in [0, \omega]} \{|f(t)|\}, \quad \bar{f} = \frac{1}{\omega} \int_0^\omega f(t) dt, \quad \hat{f} = \frac{1}{\omega} \int_0^\omega |f(t)| dt$$

and

$$Q(v) = \frac{1}{\omega} \int_0^\omega q(t, v) dt,$$

where $f(t)$ is a continuous ω -periodic function, $q(t, v)$ is a continuous function and ω -periodic with respect to t .

In this paper, we always make the following assumptions for system (1.5).

- (H₁) $\rho > 0, \sigma_1, \sigma_2$ are three constants. $\tau_1(t), \tau_2(t), r(t), a(t)$ and $d(t)$ are continuous ω -periodic functions. In addition, $\int_0^\omega r(t) dt > 0, \int_0^\omega d(t) dt > 0$ and $a(t) > 0$ for any $t \in [0, \omega]$;
- (H₂) $\phi(t, v)$ and $h(t, v)$ are continuous functions and ω -periodic with respect to t . $\phi_v(t, v)$ and $h_v(t, v)$ are also continuous functions.

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