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Melnikov's method for a general nonlinear vibro-impact oscillator ☆

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Melnikov's method is presented for a general nonlinear vibro-impact oscillator, and Melnikov function for homoclinic orbits is obtained analytically. A Duffing vibro-impact oscillator is given to illustrate the application of the procedures. The obtained results are verified by the phase portrait, Poincaré surface of section and bifurcation diagrams.

1. Introduction

In recent years, a considerable amount of research activity has focused on non-smooth dynamical systems, including vibro-impact systems [1–3], collision dynamics [4], electronic circuits [5] and stick-slip motions [6] and so on. These systems often exhibit very complicated dynamics, such as periodic-adding cascades [7,8], and so called non-smooth bifurcations [9–13] (including grazing bifurcations, border-collision bifurcations and sliding bifurcations).

By calculating the distance between the stable and unstable manifold, Melnikov's method [14] is a powerful approximate tool for investigating chaos occurrence in near Hamiltonian systems, and has been successfully applied to the analysis of chaos in smooth systems. To our knowledge, the existence of a simple zero of the Melnikov function implies the possible occurrence of chaos in the sense of Smale horseshoes. In particular, for planar non-smooth systems, Chow and Shaw [15,16] explored the bifurcations and chaos in linear systems with impacts via Melnikov's method. Kunze [17] and Kukucka [18] discussed the Melnikov function for some special planar non-smooth systems without sliding. But the procedure proposed in Refs. [15–18] is only valid for piecewise linear systems. Du and Zhang [19] proposed Melnikov's method for nonlinear impact oscillators. Awrejcewicz [20–23] discussed the chaos prediction in non-smooth systems with sliding, by the derived Melnikov function.

In the paper, based on the heuristic level in Ref. [17], we constructed the Melnikov function for homoclinic orbits in a general nonlinear vibro-impact oscillator, and the obtained analytical conditions of the threshold of chaos are verified by some numerical results. This paper is organized as follows. In Section 2, the Melnikov function for the homoclinic orbits is constructed. In Section 3, as a typical example, a double-well Duffing vibro-impact oscillator is considered. Moreover, the analytical results are verified numerically. At last, the conclusions are given.

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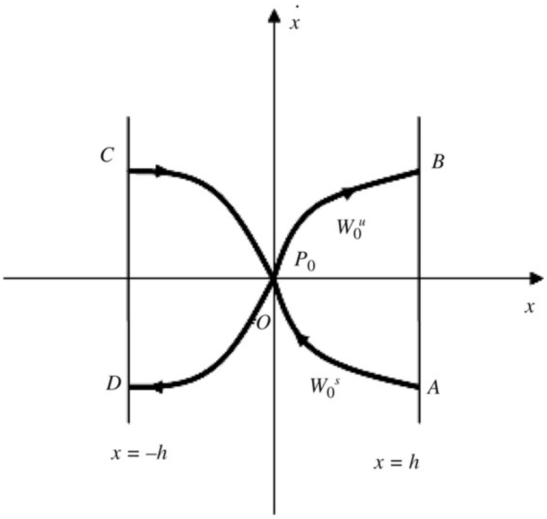

Fig. 1. The homoclinic orbits of the unperturbed system.

2. Melnikov function for homoclinic orbits

A general nonlinear vibro-impact oscillator is considered. The model is nonlinear oscillators with constraints which lead to motions with impacts. The motion of the oscillator between impacts is governed as follows:

$$\ddot{x} = f(x) + \varepsilon g(x, \dot{x}, t), \quad |x| < h \tag{1}$$

whereas the impact is assumed as instantaneous, and the impact law is

$$\dot{x}_+ = -r\dot{x}_-, \quad |x| = h, \tag{2}$$

where h is the nonnegative constant, and the r is the restitution coefficient depicting the impact energy losses, subscripts "−" and "+" denote the instants just before and after impacts. The left and right boundary functions are defined as

$$H(x, \dot{x}) = |x| - h. \tag{3}$$

First we take $r = 1 - \varepsilon r_0 \in (0, 1]$ in order to make use of the globally computable solutions of the unperturbed integrable system. The system governed by Eqs. (1) and (2) can be rewritten in the following form:

$$maineqLabel(4)\dot{X} = F(X) + \varepsilon G(X, t), \qquad H(X) < 0, \tag{4a}$$

$$X_+ = P * X_-, \qquad H(X) = 0, \tag{4b}$$

where

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \qquad P = \begin{bmatrix} 1 & 0 \\ 0 & -(1 - \varepsilon r_0) \end{bmatrix}, \qquad F(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \end{bmatrix}, \qquad G(X, t) = \begin{bmatrix} g_1(X, t) \\ g_2(X, t) \end{bmatrix},$$

$$f_1(X) = X_2, \qquad f_2(X) = f(X), \qquad g_1(X, t) = 0, \qquad g_2(X, t) = g(X, t).$$

Based on the Melnikov's theory in Ref. [14], we outline the following similar assumptions for the system (4):

(A1) The functions F and G are sufficiently smooth (C^r, $r > 1$) and bounded, and G is T-periodic function in t.
(A2) For $\varepsilon = 0$ in the system (4), we assume that the unperturbed system is a Hamiltonian one and possesses a homoclinic orbit $X_h(t)$ connecting the hyperbolic saddle point $p_0 = (0, 0)$.

By the assumption (A2), the unperturbed system (i.e. $\varepsilon = 0$ in Eqs. (4)) possesses a unique saddle equilibrium $p_0(0, 0)$, two homoclinic orbits $\Gamma_+^0 = AOB$ and $\Gamma_-^0 = COD$, which are shown in Fig. 1. The corresponding arrowheads represent the direction of stable ($= s$) and unstable ($= u$) manifolds W_0^s, W_0^u of p_0.

Equivalently, the system (4) can be suspended as follows:

$$maineqLabel(5)\dot{X} = F(X) + \varepsilon G(X, \theta), \qquad H(X) < 0, \tag{5a}$$

$$X_+ = P * X_-, \qquad H(X) = 0, \quad \dot{\theta} = 1, \tag{5b}$$

where $\theta = t \pmod T$. The Poincaré section is taken as

$$\Sigma_{t_0} = \left\{(\theta, X) \in S^1 \times R^2 | \theta = t_0 \in [0, T]\right\}. \tag{6}$$