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Parallel simultaneous stabilization of two systems governed by partial differential equations subject to actuator saturation

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ABSTRACT

A parallel simultaneous stabilization of two systems governed by partial differential equation (PDE), with conservation law subject to actuator saturation, is considered, and a method on the control design is proposed. Based on the orthogonal decomposition for a Port-Controlled Hamiltonian system, an abstract approach to the parallel simultaneous stabilization of two systems governed by partial differential equation, with conservation law subject to actuator saturation, is established. The study shows that the parallel simultaneous stabilization controller obtained in this paper is valid in the case of the systems governed by the PDEs.

1. Introduction

Considerable attention has been given to the study of the control problem of the system with saturation phenomena, because almost all systems in practice are constrained by the limits of capacity of their physical components. Such a constraint exists in different parts of a practical system, such as in actuators and sensors. Among all the saturation phenomena, one of the most important is the actuator saturation (AS), which has been widely studied since the 1950s and for which many significant results have been proposed in last decade [1–3]. Recently, a kind of important approach to the stability analysis and control design was developed for dynamics with AS [4,5]. Energy-based control and stability analysis have been extensively studied for a wide range of physical systems, including robotic systems, etc. — see, e.g. [6]. In recent years, port-controlled Hamiltonian systems have been thoroughly investigated in a series of works [6–10]. The Hamiltonian function, the sum of potential energy and kinetic energy in physical systems, is a good candidate of Lyapunov functions for many physical systems. Due to this and its nice structural in practical control designs, up to now, the energy-based approach has been used in various control problems. In practical control designs, due to systems' uncertainty, failure modes or systems with various modes of operation, the Parallel Simultaneous Stabilization problem, which is one of the important research topics in the area of robust control frequently, has to be studied,and has drawn considerable attention. Almost all of the results have been obtained for the linear systems. Recently, the PSS problem for the Hamiltonian systems was investigated, and some good results were derived for the existence of the PSS controllers, of a set of the Hamiltonian systems [11].

As far as the parallel simultaneous stabilization of the two systems governed by the PDEs is concerned, there are seldom results. The aim of the present paper is to investigate the parallel simultaneous stabilization of the Camassa–Holm (CH) equation and Degasperis–Procesi (DP) equation, subject to actuator saturation.

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In 1999, Degasperis and Procesi [12] showed, using the method of asymptotic integrability, that the PDE

$$u_t - u_{xxt} + (b+1)uu_x = bu_x u_{xx} + uu_{xxx} \tag{1.1}$$

cannot be completely integrable unless $b = 2$ or $b = 3$. The case $b = 2$ is the following Camassa–Holm (CH) shallow water equation (see [13], also [14])

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx} \tag{1.2}$$

which is well known to be integrable [15,16] and to possess multi-peakon solutions as follows:

$$u(x,t) = \sum_{j=1}^{N} p_j(t) e^{-|x-q_j(t)|}, \tag{1.3}$$

which is driven from the peakon solution $u_c(x,t) = ce^{-|x-ct|}$ originally found by Camassa and Holm [13] and which replicates a feature that is characteristic for the waves of great height-waves of largest amplitude, that are exact solutions of the governing equations for water waves (cf. [17–19]). Moreover, these solutions are orbitally stable, that is, their shape is stable under small perturbations and therefore these waves are physically recognizable — see the papers [20–22]. Also see the paper [23] for trains of solitary waves. The N-peakon solutions (1.3) are weak solutions with discontinuous first derivatives at the positions q_j of the peaks; q_j, p_j are canonical coordinates and moments of an integrable finite-dimensional Hamiltonian system [24,25]. The case $b = 3$ is the Degasperis–Procesi (DP) equation, which may be written in dispersionless form, as follows:

$$u_t - u_{xxt} + 4uu_x = 3u_x u_{xx} + uu_{xxx} \tag{1.4}$$

can be written as a system for $u(x,t)$ and $m(x,t)$

$$\begin{cases} m_t + m_x u_t + 3mu_x = 0, \\ m = u - u_{xx}. \end{cases} \tag{1.5}$$

Degasperis, Holm and Hone [26–28] proved that Eq. (1.4) is integrable by constructing its lax pair

$$(\partial_x - \partial_x^3)\varphi = \lambda m\varphi; \; \varphi_t = \left[\frac{1}{\lambda}(c - \partial_x^2) + u_x - u\partial_x\right]\varphi \tag{1.6}$$

and admits multi-peakon solutions, and explained the connection with a negative flow in the Kaup–Kupershmidt hierarchy via a reciprocal transformation. Lundmark and Szmigielski [29] presented an inverse scattering approach for computing n-peakon solutions of the DP equation, which has the following form:

$$u(x,t) = \sum_{k=1}^{N} m_k(t) e^{-|x-x_k(t)|}, \qquad m(x,t) = \sum_{k=1}^{N} 2m_k(t)\delta(x - x_k(t)) \tag{1.7}$$

where δ is the Dirac delta distribution. Zhou [30] discussed the blow-up phenomenon of Eq. (1.4) and proved the global existence of the solution. Yin [31,32] studied the Cauchy problem of Eq. (1.4), gave an explosion criterion of strong solutions with odd initial data,and proved the existence of global strong solutions and global weak solutions. Although the DP equation (1.4) has a similar form to the CH equation (1.2) and admits exact peakon solutions analogous to the Camassa–Holm peakons, the two equations are pretty different. For two equations, the different isospectral problem and the fact that there is no simple transformation of Eq. (1.4) into Eq. (1.2) imply that Eq. (1.4) is different from Eq. (1.2) in the integrable structures and the form of the conservation laws, and we are also not aware of a geometric interpretation of Eq. (1.4) while Eq. (1.2) is a re-expression of geodesic flow on the diffeomorphism group of the circle/line (cf. [33,34]). Despite these shortcomings, the DP equation (1.4) is still very interesting as it is a new integrable shallow water equation, and presents quite a rich structure. The investigation of traveling wave solutions plays an important role in understanding the physical phenomena of nonlinear systems. Tsuchida et al. [35] studied a system of semi-discrete coupled nonlinear Schrödinger equations, and constructed the soliton solutions by the extension of the inverse scattering method. Tian et al. [36] discussed the traveling wave solutions and double soliton solutions for CH equation (1.2), and introduced the definitions of concave, convex peaked soliton and smooth soliton solutions. Vakhnenko et al. [37] investigated the traveling wave solutions of the DP equation by using the transformed version of the DP equation which is a generalization of the Vaknenko equation. It is well known that the parameters greatly influence the types of the solution of the system. Bifurcation analysis has been introduced to understand the effect of the parameters on the solutions of the system. Guo et al. [38] employed the bifurcation method of the phase portraits to study periodic cusp wave solutions and single solutions for Eq. (1.1). Liu et al. [39–42] studied the peakons and the periodic cusp wave solutions for the generalized CH equation and CH–c equation by using the bifurcation method of dynamical systems.