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Periodic solutions for p-Laplacian neutral Rayleigh equation with a deviating argument

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Abstract

By using topological degree theory and some analytical skill, some criteria to guarantee the existence of ω-periodic solutions are derived for p-Laplacian neutral Rayleigh equation with a deviating argument of the following form

$$\left(\phi_p\left((x(t)-cx(t-\sigma))'\right)\right)'+f(x'(t))+g(x(t-\tau(t)))=e(t).$$

MSC: 34C25

Keywords: Periodic solutions; Neutral Rayleigh equation; Deviating argument; p-Laplacian; Topological degree

1. Introduction

Throughout this paper, $1<p<\infty$ is a fixed real number. The conjugate exponent of p is denoted by q, i.e. $\frac{1}{p}+\frac{1}{q}=1$. Let $\phi_p:\mathbf{R}\to\mathbf{R}$ be the mapping defined by $\phi_p(u)=|u|^{p-2}u$. Then ϕ_p is a homeomorphism of $\mathbf{R}$ with the inverse $\phi_q(u)=|u|^{q-2}u$. In this paper, we will consider the existence of periodic solutions for the following neutral Rayleigh equation with a deviating argument

$$\left(\phi_p\left((x(t)-cx(t-\sigma))'\right)\right)'+f(x'(t))+g(x(t-\tau(t)))=e(t), \tag{1.1}$$

where f, g, e and τ are real continuous functions on $\mathbf{R}$, τ and e are periodic with period ω, $\omega>0$ is fixed, $c,\sigma\in\mathbf{R}$ are constants such that $|c|\neq 1$.

In recent years, the existence of periodic solutions for second-order Rayleigh equations with a deviating argument

$$x''(t)+f(x'(t))+g(t,x(t-\tau(t)))=e(t), \tag{1.2}$$

has been extensively studied in the literature, we refer the readers to [1–4] and the references cited therein. In [5–7], the problems on the existence of periodic solutions for Rayleigh equations of p-Laplacian type

$$(\phi_p(x'(t)))'+f(x'(t))+g(x(t-\tau(t)))=e(t), \tag{1.3}$$

were also discussed by using Mahwin's coincidence degree theory [8].

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In [9], Zonga and Liang consider the following equation

$$(\phi_p(x'(t)))' + f(t, x'(t-\sigma(t))) + g(t, x(t-\tau(t))) = e(t), \tag{1.4}$$

where $f, g \in C(\mathbf{R}\times\mathbf{R}, \mathbf{R})$ and f, g are 2π-periodic with their first arguments, σ, τ and e are continuously 2π-periodic functions defined on $\mathbf{R}$. Under the assumptions of $f(t, 0) = 0$ and $\int_0^{2\pi} e(t)\mathrm{d}t = 0$, they obtained the following result.

Theorem A ([9]). *Suppose there exist positive constants K, d, M such that*
(H_1) $|f(t, x)| \le K$ *for* $(t, x) \in \mathbf{R}\times\mathbf{R}$;
(H_2) $xg(t, x) > 0$ *and* $|g(t, x)| > K$ *for* $|x| > d$ *and* $t \in \mathbf{R}$;
(H_3) $g(t, x) \ge -M$, *for* $x \le -d$ *and* $t \in \mathbf{R}$.
Then (1.4) *has at least one solution with period* 2π.

For a kind of second-order neutral Rayleigh functional differential equation

$$(x(t) + cx(t-r))'' + f(x'(t)) + g(t, x(t-\tau(t))) = p(t), \tag{1.5}$$

Lu and Ge [10] gave some existence theorems of 2π-periodic solutions for $|c| \ne 1$.

Very recently, Zhu and Lu [11] discussed the existence of periodic solutions for p-Laplacian neutral functional differential equation with deviating argument when $p > 2$

$$(\phi_p(x(t) - cx(t-\sigma))')' + g(t, x(t-\tau(t))) = p(t). \tag{1.6}$$

They obtained some results by translating (1.6) into a two-dimensional system to which Mawhin's continuation theorem was applied.

But for (1.1), the methods to obtain a priori bounds of periodic solutions in [11] cannot be applied to the present paper, since the crucial step $\int_0^T [g(t, x(t-\tau(t))) - p(t)]\mathrm{d}t = 0$ is no longer valid for Eq. (1.1). The purpose of this paper is to establish some criteria to guarantee the existence of ω-periodic solutions of (1.1) for any $p > 1$ by using the following Lemma 2.6 in Section 2 which is similar to Theorem 3.1 in [12] obtained by using Leray–Schauder degree theory. The significance of this paper is that even if for $c = 0$, the results are different from the corresponding ones of [5–7].

2. Preliminaries

Let $C_\omega = \{x : x \in C(\mathbf{R}, \mathbf{R}), x(t+\omega) \equiv x(t)\}$ with norm $\|x\|_\infty = \max_{t\in[0,\omega]} |x(t)|$, $C^1_\omega = \{x : x \in C^1(\mathbf{R}, \mathbf{R}), x(t+\omega) \equiv x(t)\}$ with norm $\|x\| = \max\{\|x\|_\infty, \|x'\|_\infty\}$. Clearly, C_ω and C^1_ω are Banach spaces. In what follows, we will use $\|\cdot\|_p$ to denote the L^p-norm in C_ω, i.e. $\|x\|_p = \left(\int_0^\omega |x(t)|^p\mathrm{d}t\right)^{\frac{1}{p}}$. We also define a linear operator A as follows

$$A : C_\omega \to C_\omega, \quad (Ax)(t) = x(t) - cx(t-\sigma).$$

Lemma 2.1 ([13–15]). *If $|c| \ne 1$, then A has continuous bounded inverse on C_ω, and*
(1) $\|A^{-1}x\|_\infty \le \frac{\|x\|_\infty}{|1-|c||}$, $\forall x \in C_\omega$,
(2)

$$(A^{-1}x)(t) = \begin{cases} \sum_{j\ge 0} c^j x(t-j\sigma), & |c| < 1 \\ -\sum_{j\ge 1} c^{-j} x(t+j\sigma), & |c| > 1. \end{cases} \tag{2.1}$$

(3) $\int_0^\omega |(A^{-1}x)(t)|\mathrm{d}t \le \frac{1}{|1-|c||}\int_0^\omega |x(t)|\mathrm{d}t$, $\forall x \in C_\omega$.

Lemma 2.2. *For $a_i, x_i \ge 0$, and $\sum_{i=1}^n a_i = 1$, the following inequality holds for any $p > 1$*

$$\left(\sum_{i=1}^n a_i x_i\right)^p \le \sum_{i=1}^n a_i x_i^p.$$