Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis 69 (2008) 1768–1774
Nonlinear Analysis
www.elsevier.com/locate/na

Chaotifing a linear hyperbolic system of partial differential equations by means of nonlinear boundary reflection

Yonglu Shu[1]

Department of Mathematics, Chongqing University, 400044, PR China

Received 13 June 2007; accepted 17 July 2007

Abstract

We introduced a hyperbolic linear system of partial differential equations with nonlinear boundary conditions, and proved strictly that it is chaotic under some simple assumptions. This can be understood as chaotifing a hyperbolic linear system of partial differential equations with nonlinear boundary conditions. Several examples of simulation are presented. The numerical simulation fits the theoretical analysis well.

Keywords: Chaos; Chaotifing; Hyperbolic linear system; Partial differential equation; Nonlinear boundary reflection

1. Introduction

In the literature there are plenty of examples of dynamical systems that exhibit chaotic behavior. Through numerical simulations, chaos has been shown to exist in many second-order ordinary differential equations arising from nonlinear vibrating springs and electronic circuits [1]. The mathematical justifications required in rigorously establishing the occurrence of chaos are technically very challenging [2]. While important progress in the development of mathematical chaos theory for nonlinear second-order ordinary differential equations has been made, the theory of chaos in partial differential equations has not been well developed. Unlike chaos in finite-dimensional systems, the area of chaos in partial differential equations has been a long standing open field. Mathematically, studies on infinite-dimensional systems pose much more challenging problems. For example, as phase spaces, Banach spaces possess much more structure than Euclidean spaces.

Since the early work of Holmes and Marsden [3], Knoblock, Moore, Toomre, Weiss [4], many scientists and engineers have done a lot of exploration in the area of chaos analysis for partial differential equations. Some methods of chaos analysis for ordinary differential equations have been generalized to certain kinds of partial differential equations [5]: (i) The Lyapunov exponent method, mean Lyapunov exponent and local Lyapunov exponent are introduced to characterize the chaos in systems described by partial differential equations [6,7]. It has been shown that the mean Lyapunov exponent is related to the disorder of spatial patterns, while the local Lyapunov exponent, which is a finite time average of the mean Lyapunov exponent, has a close relation to spatiotemporal patterns. (ii)

E-mail address: Shuyonglu@163.com.
[1] Foundation item: the National Natural Youth Foundation of China (0208002321018).

0362-546X/$ - see front matter
doi:10.1016/j.na.2007.07.024

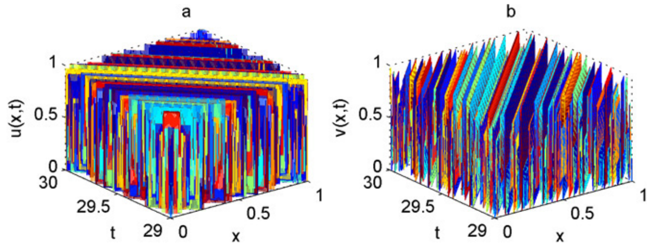

Fig. 1. a. Spatiotemporal evolution wave form of $u(x,t)$. b. Spatiotemporal evolution wave form of $v(x,t)$.

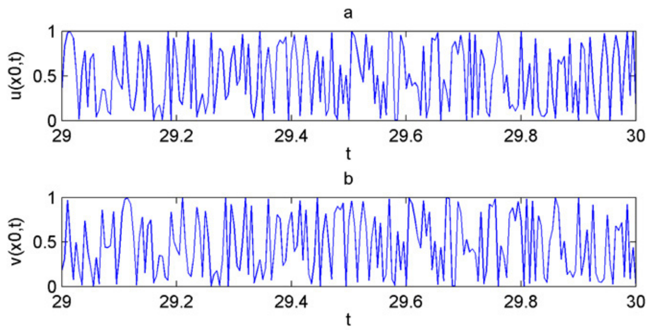

Fig. 2. a. Temporal evolution wave form of $u(x_0,t)$, $x_0 = 0.1$. b. Temporal evolution wave form of $v(x_0,t)$, $x_0 = 0.1$.

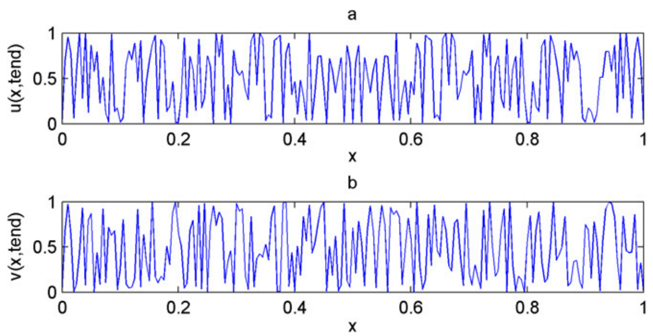

Fig. 3. a. Spatial evolution wave form of $u(x, \text{tend})$. b. Spatial evolution wave form of $v(x, \text{tend})$. tend = 30.

Measurement of chaos by means of entropy, spatial entropy has been introduced to characterize the spatial chaos for the Fisher–Kolmogorov equation with piecewise linear cubic-like nonlinearity [8]. (iii) Perturbation theory and the Melnikov method, some results for partial differential equations, in particular for nonlinear Schrödinger systems, the KdV equation and the Euler equation, have been obtained [2,9].

As pointed out by [10]: "since partial differential equations have many different types, it is not easy to give a precise definition of chaos that is uniformly meaningful for a majority of partial differential equations. Thus, chaos for partial differential equations may have to be studied on a case-by-case basis". The "history" of exploration of chaos analysis for partial differential equations clearly indicated this. Blyuss [11] use one-dimensional nonlinear Schrödinger equations to illustrate chaotic and turbulent behavior of nonlinear dispersive waves. Cai and McLaughlin [12] consider the properties of nonlinear waves and solitons of the Korteweg–de Vries equation in the presence of external perturbation. It induced chaotic behaviors in long-period waves and solitons. Cabral and Rosa [13] were the first to obtain the onset of temporal chaos for a damped and forced KdV case. The attractor for a damped and forced Korteweg–de Vries equation is studied. For large damping, it is shown that the attractor is trivial, with all solutions converging to a unique fixed point. For small damping, however, the dynamics can be complicated. Li [2] developed a systematic program for proving the existence of homoclinic orbits for perturbed soliton equations, and presented a survey on chaos in partial differential equations [9]. The author classifies soliton equations into three categories: