Nonlinear Analysis 70 (2009) 4274–4293

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Infinitely many nonoscillatory solutions for second order nonlinear neutral delay differential equations

Zeqing Liu [a,*], Shin Min Kang [b]

[a] Department of Mathematics, Liaoning Normal University, P. O. Box 200, Dalian, Liaoning 116029, People's Republic of China

[b] Department of Mathematics and The Research Institute of Natural Science, Gyeongsang National University, Chinju 660-701, Republic of Korea

ARTICLE INFO

Article history:
Received 29 August 2006
Accepted 29 September 2008

MSC:
34K15
34C10

Keywords:
Second order nonlinear neutral delay differential equation
Infinitely many nonoscillatory solutions
Contraction mapping
Mann iterative sequence with errors

ABSTRACT

In this paper we consider the second order nonlinear neutral delay differential equation

$$\left[a(t)\,(x(t)+b(t)x(t-\tau))'\right]' + [h(t,x(h_1(t)),x(h_2(t)),\ldots,x(h_k(t)))]'$$
$$+f(t,x(f_1(t)),x(f_2(t)),\ldots,x(f_k(t))) = g(t), \quad t \ge t_0,$$

where $\tau > 0$, $a, b, g \in C([t_0,+\infty),\mathbb{R})$ with $a(t) > 0$ for $t \ge t_0$, $h \in C^1([t_0,+\infty)\times\mathbb{R}^k,\mathbb{R})$, $f \in C([t_0,+\infty)\times\mathbb{R}^k,\mathbb{R})$, $h_l \in C^1([t_0,+\infty),\mathbb{R})$ and $f_l \in C([t_0,+\infty),\mathbb{R})$ with

$$\lim_{t\to+\infty} h_l(t) = \lim_{t\to+\infty} f_l(t) = +\infty, \quad l = 1,\ldots,k.$$

Under suitable conditions, by making use of the Banach fixed point theorem, we show the existence of infinitely many nonoscillatory solutions, which are uncountable, for the above equation, suggest several Mann type iterative approximation sequences with errors for these nonoscillatory solutions and establish some error estimates between the approximate solutions and the nonoscillatory solutions. Five nontrivial examples are given to illustrate the advantages of our results.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction and preliminaries

In recent years there has been much research activity concerning the oscillation and nonoscillation of solutions for various kinds of second order neutral delay differential equations, for example, see [1–10,12–14]. Huang [9] and Elbert [5] established a few oscillation and nonoscillation criteria for the second order linear differential equation

$$x''(t) + q(t)x(t) = 0, \quad t \ge 0, \tag{1.1}$$

where $q \in C([0,+\infty),\mathbb{R}^+)$. Tang and Liu [12] studied the existence of bounded oscillation for the second order linear delay differential equation of unstable type

$$x''(t) = p(t)x(t-\tau), \quad t \ge t_0, \tag{1.2}$$

where $\tau > 0$, $p \in C([t_0,+\infty),\mathbb{R}^+)$ and $p(t) \not\equiv 0$ on any interval of length τ. Agarwal, O'Regan and Saker [3] investigated some oscillation criteria for the second order nonlinear neutral delay dynamic equation

$$\left[r(t)((x(t)+P(t)x(t-\tau))^{\triangle})^{\gamma}\right]^{\triangle} + f(t,x(t-\delta)) = 0, \quad t \ge t_0 \tag{1.3}$$

* Corresponding author.
E-mail addresses: zeqingliu@dl.cn (Z. Liu), smkang@nongae.gsnu.ac.kr (S.M. Kang).

0362-546X/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2008.09.013

on a time scale $\mathbb{T}$, where γ is a quotient of odd positive integers with $r(t)$ and $p(t)$ real-valued positive functions defined on $\mathbb{T}$. In 1998, Kulenović and Hadžiomerspahić [10] discussed the second order linear neutral delay differential equation with positive and negative coefficients

$$\frac{d^2}{dt^2}[x(t)+cx(t-\tau)]+Q_1(t)x(t-\sigma_1)-Q_2(t)x(t-\sigma_2)=0, \quad t\ge t_0, \tag{1.4}$$

where $c\in\mathbb{R}$, $\tau>0$, $\sigma_1,\sigma_2\in[0,+\infty)$, $Q_1,Q_2\in C([t_0,+\infty),\mathbb{R}^+)$, and under the conditions $c\ne\pm1$, $aQ_1(t)\ge Q_2(t)$, $t\ge t_0$ and other conditions, they gave some sufficient conditions for the existence of a nonoscillatory solution of Eq. (1.4). In 2004, Cheng and Annie [4] continued to investigate a few sufficient conditions, which guarantee the existence of a nonoscillatory solution for Eq. (1.4) by omitting the conditions $c\ne 1$ and $aQ_1(t)\ge Q_2(t)$, $t\ge t_0$, which were used by Kulenović and Hadžiomerspahić [10]. In 2005, Yu and Wang [13] studied the existence of a nonoscillatory solution for the second order nonlinear neutral delay differential equations with positive and negative coefficients

$$\left[r(t)\left(x(t)+P(t)x(t-\tau)\right)'\right]'+Q_1(t)f(x(t-\sigma_1))-Q_2(t)g(x(t-\sigma_2))=0, \quad t\ge t_0, \tag{1.5}$$

where $\tau>0$, $\sigma_1,\sigma_2\in[0,+\infty)$, $P,Q_1,Q_2,r\in C([t_0,+\infty),\mathbb{R})$, $f,g\in C(\mathbb{R},\mathbb{R})$. However, the results in [4,10,13] only dealt with the existence of a nonoscillatory solution of Eqs. (1.4) and (1.5), respectively, and did not suggest the iterative approximations of the nonoscillatory solution and the existence of infinitely many nonoscillatory solutions of Eqs. (1.4) and (1.5).

Our aim in this paper is to investigate the following second order nonlinear neutral delay differential equation

$$\begin{aligned}&\left[a(t)\left(x(t)+b(t)x(t-\tau)\right)'\right]'+[h(t,x(h_1(t)),x(h_2(t)),\ldots,x(h_k(t)))]'\\&\quad+f(t,x(f_1(t)),x(f_2(t)),\ldots,x(f_k(t)))=g(t), \quad t\ge t_0,\end{aligned} \tag{1.6}$$

where $\tau>0$, $a,b,g\in C([t_0,+\infty),\mathbb{R})$ with $a(t)>0$ for $t\ge t_0$, $h\in C^1([t_0,+\infty)\times\mathbb{R}^k,\mathbb{R})$, $f\in C([t_0,+\infty)\times\mathbb{R}^k,\mathbb{R})$, $h_l\in C^1([t_0,+\infty),\mathbb{R})$ and $f_l\in C([t_0,+\infty),\mathbb{R})$ with

$$\lim_{t\to+\infty}h_l(t)=\lim_{t\to+\infty}f_l(t)=+\infty, \quad l=1,\ldots,k.$$

Utilizing the contraction mapping principle, we give several existence results of infinitely many nonoscillatory solutions, which are uncountable, for Eq. (1.6), construct a few Mann type iterative approximation schemes with errors for these nonoscillatory solutions and discuss error estimates between the approximate solutions and the nonoscillatory solutions. These results presented in this paper extend, improve and unify all results due to Cheng and Annie [4], Kulenović and Hadžiomerspahić [10] and Yu and Wang [13]. Five nontrivial examples are considered to illustrate our results.

By a solution of Eq. (1.6), we mean a function $x\in C([t_1-\tau,+\infty),\mathbb{R})$ for some $t_1\ge t_0$, such that $x(t)+b(t)x(t-\tau)$ and $a(t)\left(x(t)+b(t)x(t-\tau)\right)'$ are continuously differentiable in $[t_1,+\infty)$ and such that Eq. (1.6) is satisfied for $t\ge t_1$. As is customary, a solution of Eq. (1.6) is said to be oscillatory if it has arbitrarily large zeros and nonoscillatory otherwise. It is assumed throughout this paper that:

(H) For any given constants M and N with $M>N>0$, there exist four functions $P_{M,N},Q_{M,N},R_{M,N},W_{M,N}\in C([t_0,+\infty),\mathbb{R}^+)$ satisfying

$$\begin{aligned}&|f(t,u_1,u_2,\ldots,u_k)-f(t,\bar u_1,\bar u_2,\ldots,\bar u_k)|\le P_{M,N}(t)\max\{|u_l-\bar u_l|:1\le l\le k\},\\&|h(t,u_1,u_2,\ldots,u_k)-h(t,\bar u_1,\bar u_2,\ldots,\bar u_k)|\le R_{M,N}(t)\max\{|u_l-\bar u_l|:1\le l\le k\}\\&\quad\text{for } t\in[t_0,+\infty),\ u_l,\bar u_l\in[N,M] \text{ and } \le l\le k;\end{aligned} \tag{1.7}$$

$$\begin{aligned}&|f(t,u_1,u_2,\ldots,u_k)|\le Q_{M,N}(t) \quad\text{and}\quad |h(t,u_1,u_2,\ldots,u_k)|\le W_{M,N}(t)\\&\quad\text{for } t\in[t_0,+\infty),\ u_l\in[N,M] \text{ and } 1\le l\le k;\end{aligned} \tag{1.8}$$

$$\int_{t_0}^{+\infty}A(s)\max\{P_{M,N}(s),Q_{M,N}(s),|g(s)|\}ds<+\infty \tag{1.9}$$

and

$$\int_{t_0}^{+\infty}\max\left\{\frac{R_{M,N}(s)}{a(s)},\frac{W_{M,N}(s)}{a(s)}\right\}ds<+\infty, \tag{1.10}$$

where

$$A(t)=\int_{t_0}^{t}\frac{1}{a(s)}ds, \quad t\in[t_0,+\infty).$$

Note that the function A is positive and increasing in $(t_0,+\infty)$. Let X denote the Banach space of all continuous and bounded functions on $[t_0,+\infty)$ with norm $\|x\|=\sup_{t\ge t_0}|x(t)|$, and

$$X(N,M)=\{x\in X: N\le x(t)\le M,\ t\ge t_0\} \quad\text{for } M>N>0.$$

Download English Version:

https://daneshyari.com/en/article/843056

Download Persian Version:

https://daneshyari.com/article/843056

Daneshyari.com