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Stability analysis in Lagrange sense for a non-autonomous Cohen–Grossberg neural network with mixed delays

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ABSTRACT

The paper discusses the global exponential stability in the Lagrange sense for a non-autonomous Cohen–Grossberg neural network (CGNN) with time-varying and distributed delays. The boundedness and global exponential attractivity of non-autonomous CGNN with time-varying and distributed delays are investigated by constructing appropriate Lyapunov-like functions. Moreover, we provide verifiable criteria on the basis of considering three different types of activation function, which include both bounded and unbounded activation functions. These results can be applied to analyze monostable as well as multistable biology neural networks due to making no assumptions on the number of equilibria. Meanwhile, the results obtained in this paper are more general and challenging than that of the existing references. In the end, an illustrative example is given to verify our results.

1. Introduction

Cohen–Grossberg neural networks (CGNNs) have found many applications since the pioneering work of Cohen and Grossberg [1]. In employing CGNNs to solve problems, one of the most desirable properties of CGNNs is the Lyapunov stability. From a dynamical system point of view, globally stable networks in Lyapunov sense are monostable systems, which have a unique equilibrium attracting all trajectories asymptotically, more specific results are referred to [2–10,25–29]. In many other applications, however, monostable neural networks have been found to be computationally restrictive and multistable dynamics are essential to deal with the important neural computations desired. In these circumstances, neural networks are no longer globally stable and more appropriate notions of stability are need to deal with multistable systems, especially, such as the Cohen–Grossberg neural network. When applications are taken into account in biology, it is necessary and important to deal with multistable properties. In [11], the boundedness, attractivity and complete convergence of a multistable network are investigated. Furthermore, in [13] the author studied the global exponential stability (GES) in the Lagrange sense for recurrent neural networks basing on [11,12]. It is noted that unlike Lyapunov stability, Lagrange stability refers to the stability of the total system, not the stability of the equilibrium point. Hence, the Lagrange stability is considered on the basis of the boundedness of solutions and the existence of global attractive sets. About Lagrange stability, for more results in the theory and application of dynamical systems refer to [14–21]. At present, although a series of results for CGNNs

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are obtained, the stability analysis in the Lagrange sense does not appear. So from the theoretical and application view, it is necessary to study the stable properties in the Lagrange sense for a Cohen–Grossberg neural network.

Motivated by the above discussion, our objective in this paper is to study the global exponential stability in Lagrange sense for the non-autonomous CGNN with time-varying and distributed delays. We provide verifiable criteria for the boundedness of the networks and the existence of globally exponentially attractive (GEA) sets by constructing appropriate Lyapunov-like functions. Meanwhile, we consider three different types of activation function, which include both bounded and unbounded activation functions. Hence, it is believed that the results are significant and useful for the design and applications of the non-autonomous CGNNs with mixed delays.

This paper consists of the following sections. Section 2 describes some preliminaries, including some necessary notations, definitions, assumptions and lemmas. The main results basing on three different types of activation function are obtained in Section 3. Section 4 gives a numerical example to demonstrate the main results. We conclude this paper in Section 5.

2. Preliminaries

Consider the following non-autonomous Cohen–Grossberg neural network (CGNN) with time-varying and distributed delays:

$$\dot{x}_i(t) = -a_i(t, x_i(t))\left[d_i(t, x_i(t)) - \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t)) - \sum_{j=1}^{n} c_{ij}(t)f_j(x_j(t-\tau_j(t))) - \sum_{j=1}^{n} e_{ij}(t)\int_{t-\lambda}^{t} f_j(x_j(s))\mathrm{d}s - I_i(t)\right], \quad (1)$$

where $i = 1, \dots, n$, n is the number of neurons; $x_i(t)$ is the state variable of the ith neuron at time t; $f_j(\cdot)$ denotes the activation function; $a_i(\cdot)$ denotes the amplification function and $d_i(\cdot)$ denotes the behaved function, where a_i and d_i are continuous functions on R^2; $b_{ij}(t)$, $c_{ij}(t)$ and $e_{ij}(t)$ are the weight strength of the jth unit on the ith unit at time t; $I_i(t)$ is a variable input (bias); also f_j, b_{ij}, c_{ij}, e_{ij} and I_i are continuous functions on R; $\tau_j(t)$ corresponds to the transmission delay along the axon of the jth unit; the scalar $\lambda > 0$ is the known distributed delay.

Let $h = \max(\max_{1\le j\le n}\{\tau_j\}, \lambda)$ and C is the Banach space of continuous functions $\varphi : [-h, 0] \to R^n$ with the norm $\|\varphi\| = \sup_{s\in[-h,0]} |\varphi(s)|$. For a given constant $H > 0$, C_H is defined as the subset $\{\varphi \in C : \|\varphi\| \le H\}$. Let KL denotes the set of all nonnegative continuous functions $P : C \to [0, +\infty]$, mapping bounded sets in C into bounded sets in $[0, +\infty]$. For any initial function $\varphi \in C$, the solution of (1) that starts from the initial condition φ will be denoted by $x(t; \varphi)$. If there is no need to emphasize the initial condition, any solution of the network (1) will also simply be denoted by $x(t)$.

In order to establish the conditions of main results for the neural networks (1), we have the following assumptions:

(H1) For $i = 1, 2, \dots, n$, there exist positive constants $\underline{a}_i$ and $\overline{a}_i$ such that $0 < \underline{a}_i \le a_i(t, u) \le \overline{a}_i < +\infty$ for all $t, u \in R$;

(H2) For $i = 1, 2, \dots, n$, there exist positive constants $\underline{d}_i$ and $\overline{d}_i$ such that $0 \le u d_i(t, u)$; $\underline{d}_i u \le d_i(t, u) \le \overline{d}_i u$ for all $t, u \in R$;

For convenience, we introduce some notations. We use $x = (x_1, x_2, \dots, x_n)^{\mathrm{T}} \in R^n$ to denote a column vector, in which the symbol $(^{\mathrm{T}})$ denotes the matrix transpose of a vector. $E_{n\times n}$ will be used to denote the $n \times n$ identity matrix. $\tau_j(t)$ is nonnegative and bounded, i.e. $0 \le \tau_j(t) \le \tau_j$, $\tau(t) = (\tau_1(t), \tau_2(t), \dots, \tau_n(t))^{\mathrm{T}}$ and $0 \le \dot{\tau}(t) \le \delta < 1$; Throughout the paper, we assume that $\widetilde{B}$, $\widetilde{C}$, and $\widetilde{E}$ to denote the connection weight matrices $\overline{b}_{ij}$, $\overline{c}_{ij}$, $\overline{e}_{ij}$, respectively, where

$$\overline{b}_{ij} = \max_{t\in[0,\infty]} b_{ij}(t); \qquad \overline{c}_{ij} = \max_{t\in[0,\infty]} c_{ij}(t); \qquad \overline{e}_{ij} = \max_{t\in[0,\infty]} e_{ij}(t).$$

Also we assume that

$$\overline{I}_i = \max_{t\in[0,\infty]} I_i(t); \quad i = 1, 2, \dots, n.$$

In this paper, we shall consider three classes of activation functions for the neural network model (1). To this end, we define the vector function $f \in C(R^n, R^n)$ by $f(x) = (f_1(x_1), f_2(x_2), \dots, f_n(x_n))^{\mathrm{T}}$, where $x = (x_1, x_2, \dots, x_n)^{\mathrm{T}} \in R^n$. For convenience, we define

$$\Pi := \{\psi \in C(R, R) | s\psi(s) > 0, s \ne 0, \text{ and } D^{+}\psi(s) \ge 0, s \in R\}.$$

(H3) $f(\cdot) \in \Theta$, where

$$\Theta := \{f(\cdot) | f_i \in C(R, R), \exists k_i > 0, |f_i(x_i)| \le k_i, \forall x_i \in R, i = 1, 2, \dots, n\}.$$

Obviously, (H3) consists of all bounded continuous functions, of course, it also consists of sigmoid functions. The constants $k_i (i = 1, 2, \dots, n)$ of the Θ-type activation functions will be called saturation constants. Note that for this class of activation functions, it is not required that they be monotone and $f(0) = 0$.

(H3′) $f(\cdot) \in \Upsilon$, where

$$\Upsilon := \{f(\cdot) | f_i \in \Pi, \exists k_i > 0, x_i f_i(x_i) \le k_i x_i^2, \forall x_i \in R, i = 1, 2, \dots, n\}.$$