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Some new exact solutions for MHD aligned creeping flow and heat transfer in second grade fluids by using Lie group analysis

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ABSTRACT

The Lie group analysis and the basic similarity reductions are performed for the MHD aligned slowly flowing and heat transfer in second grade fluid with neglecting the inertial terms. When the inertia terms are simply omitted from the equations of motions the resulting solutions are valid approximately for $Re \prec\prec 1$. This fact can also be deduced from the dimensionless form of the momentum and energy equations. By employing Lie group theory, the full one-parameter infinitesimal transformation group leaving the equations of motion invariant is derived along with its associated Lie algebra. Subgroups of the full group are then used to obtain a reduction by one in the number of independent variable in the system. Two new exact solutions generated from the similarity transformation to these equations, are provided. Comparisons with previously work is performed and the result is found to be inexcellent. Finally, some boundary value problems are discussed.

1. Introduction

The flow of non-Newtonian fluids has gained a lot of importance in recent years because of its numerous technological applications: including plastic manufacture, performance of lubricants, application of paints, processing of food and movement of biological fluids. Most biologically important fluids contain higher molecular weight components and are, therefore, non-Newtonian. The unusual properties of polymer melts and solutions, together with the desirable attributes of many polymeric solids, have given rise to the world wide industry of polymer processing. Geophysical applications concerning ice and magna flows are based on non-Newtonian constitutive behaviors. Due to the practical and fundamental association of these fluids to industrial problems, several authors [1–10], have studied the theory of non-Newtonian fluids in various geometrical configurations, by using perturbation techniques, numerical techniques, inverse methods or Laplace transforms, etc. Many authors [11–13] have also employed Lie group methods to investigate different features of differential equations. The Newtonian and second grade creeping flow solutions are contrasted before [14–17]. Tanner [14] stated that the Newtonian creeping flow solution is also a solution of the second grade flow for given specific boundary conditions. Huilgol [15] established the criteria under which Newtonian solutions are the only solutions available for a second grade fluid. A generalization of the theorem of Tanner was given by Fosdick and Rajagopal [16] who derived a drag formula for an immersed body in a second grade fluid. The problem of an additional boundary condition for a second grade fluid was discussed by Rajagopal [17] who considered the creeping flow over a porous flat plate. The extensions of Tanner's theorem to three-dimensional flows were discussed as well. The problems arising in having additional boundary conditions in the case of differential type fluids were reviewed by Dunn and Rajagopal [18]. A similarity solution for non-parallel porous walls was developed by Bourgin and Tichy [19]. The slow motion of a body in a second grade fluid was investigated by Galdi

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and Rajagopal [20]. In all of the above mentioned studies the aligned magnetohydrodynamic fluid is negligible. However, the interest in MHD fluid flows stems from the fact that liquid metals that occur in nature and industry are electrically conducting. These fluids, for the most part, are of finite electrical conductivity. Also considered are MHD fluid flows of infinite conductivity. These two types of fluid flow are attractive both from a mathematical as well as a physical standpoint. Many important fields of applications is the electromagnetic propulsion. Basically an electromagnetic propulsion system consists of power source, such as a nuclear reactor, plasma, and a tube through which the plasma is accelerated by electromagnetic forces. The study of such systems, which is closely associated with magnetochemistry, requires a complete understanding of the equation of state and transport properties such as diffusion, shear stress–shear rate relationship, thermal conductivity, electrical conductivity and radiation. Some of these properties will undoubtedly be influenced by the presence of an external magnetic field which sets the plasma in hydrodynamic motion. Many researchers [21–23], investigated MHD aligned fluid for Newtonian and non-Newtonian fluids. In this work, we consider the steady plane MHD aligned creeping flow and heat transfer equations of a second grade fluid. Lie group theory [24] is applied to the equations of motion. The symmetries of the equations are found. The equations admit eight Lie point symmetries. To obtain a group invariant solution we use translation symmetries i.e. translation in x and y coordinates are used to transform the partial differential system into an ordinary differential system. Exact analytical solutions of exponential and polynomial types are found for this case. Finally, boundary value problems are obtained.

2. Formulation of the problem

The motion of the steady MHD aligned creeping flow and heat transfer equations of a second grade fluid is governed by the system equations

$$\operatorname{div}\vec{V} = 0 \tag{1}$$

$$\rho\left(\vec{V}\cdot\operatorname{grad}\right)\vec{V} = \operatorname{div}T + \rho f^{\rightarrow} + \mu_2\left(\operatorname{curl}\vec{H}\right)\times\vec{H}, \tag{2}$$

$$\operatorname{div}\vec{H} = 0, \tag{3}$$

$$\operatorname{curl}\left(\vec{V}\times\vec{H}\right) - \frac{1}{\mu_2\sigma}\operatorname{curl}\left(\operatorname{curl}\vec{H}\right) = 0 \tag{4}$$

$$\mathrm{T.L} - \operatorname{div} q + \rho r = \rho\left(\vec{V}\cdot\operatorname{grad}\right)e. \tag{5}$$

The constitutive equation for the stress of a second grade fluid [26].

$$T = -pI + \mu_{1A_1} + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{6}$$

where

$$A_1 = \operatorname{grad}\vec{V} + (\operatorname{grad}\mathrm{V}^{\rightarrow})^T$$

$$A_2 = \frac{\mathrm{d}A_1}{\mathrm{d}t} + A_1\left(\operatorname{grad}\vec{V}\right) + \left(\operatorname{grad}\vec{V}\right)^T A_1.$$

Here, $\vec{V}$ is the velocity vector, $\vec{b}$ the body force per unit mass, $\vec{H}$ the magnetic field intensity vector, P the pressure, ρ the fluid density, μ_2 the magnetic permeability, σ the electric conductivity, μ_1 the constant viscosity, α_1 and α_2 are the constant normal stress moduli, A_1 and A_2 are the first two Rivlin–Ericksen tensors, r is the radiant heating (assumed to be zero), $e = C_pT$ is the specific internal energy, where C_p is the specific heat and T is the temperature, $q = -k_1\nabla T$ is the heat flux vector, where k_1 is the constant thermal conductivity. The solenoidal equation shows that there is no magnetic pole in the flow field. Following (Refs. [27,28]), we assume

$$\mu_1 \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0. \tag{7}$$

In this paper, we shall consider a second grade fluid which undergoes isochoric motion in a plane, and in which the body force is negligible. On taking $\vec{V} = (u'(x,y), v'(x,y), 0)$, $T' = T'(x,y)$, $\vec{H} = (H_1'(x,y), H_2'(x,y), 0)$, $f = 0$, and $P' = P'(x,y)$. Under the above assumptions, the governing equations can be written as:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{8}$$

$$\begin{aligned} u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = {} & -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + \frac{\mu_1}{\rho}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) + \frac{\alpha_1}{\rho}\begin{pmatrix} 5\dfrac{\partial u'}{\partial x'}\dfrac{\partial^2 u'}{\partial x'^2} + \dfrac{\partial u'}{\partial x'}\dfrac{\partial^2 u'}{\partial y'^2} + u'\dfrac{\partial^3 u'}{\partial x'^3} + v'\dfrac{\partial^3 u'}{\partial y'^3} \\ +u'\dfrac{\partial^3 u'}{\partial x'\partial y'^2} + 2\dfrac{\partial v'}{\partial x'}\dfrac{\partial^2 v'}{\partial x'^2} + \dfrac{\partial u'}{\partial y'}\dfrac{\partial^2 u'}{\partial x'\partial y'} \\ +\dfrac{\partial u'}{\partial y'}\dfrac{\partial^2 v'}{\partial x'^2} + v'\dfrac{\partial^3 u'}{\partial y'\partial x'^2} \end{pmatrix} \\ & - \frac{\mu_2}{\rho}H'_2\left(\frac{\partial H'_2}{\partial x'} - \frac{\partial H'_1}{\partial y'}\right) \end{aligned} \tag{9}$$