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Lie symmetry analysis and exact solutions for the short pulse equation[☆]

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ABSTRACT

In this paper, the Lie symmetry analysis and the generalized symmetry method are performed for a short pulse equation (SPE). The symmetries for this equation are given. For the traveling wave solutions, the exact parametric representations are investigated. To guarantee the existence of the above solutions, all parameter conditions are determined. Furthermore, the exact analytic solutions are obtained by using the power series method.
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1. Introduction

Nonlinear partial differential equations (PDEs) arising in many physical fields like the condense matter physics, fluid mechanics, plasma physics and optics, etc, exhibit a rich variety of nonlinear phenomena. Recently, many PDEs generated from the systems of impulse and neural networks as well. The investigation of the exact solutions plays an important role in the study of nonlinear physical systems and such neural networks. A wealth of methods have been developed to find these exact solutions of a PDE though it is rather difficult. Some of the most important methods are the inverse scattering method [1], Darboux and Bäcklund transformations [2], Hirota's bilinear method [2–4], Lie symmetry analysis [5–8], CK method [9,10], etc. It is well-known that the Lie group method is a powerful and direct approach to construct exact solutions of nonlinear differential equations. Furthermore, based on the Lie group method, many other type of exact solutions of PDE can be obtained, such as the traveling wave solutions, soliton solutions, fundamental solutions [11,12], and so on.

In this paper, we will consider the short pulse equation (SPE) which has the general form

$$u_{xt} = \alpha u + \frac{1}{3}\beta(u^3)_{xx}, \tag{1}$$

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where $u = u(x, t)$ is the unknown real function and subscripts denote differentiation, $x, t \in R$, α and β are real parameters, $\alpha\beta \neq 0$. In practical, Eq. (1) can be written as the more usual form

$$u_{xt} = \alpha u + 2\beta u u_x^2 + \beta u^2 u_{xx}. \tag{2}$$

This general SPE was derived by T.Schäfer and C.E.Wayne as a model equation describing the propagation of ultra-short light pulses in silica optical fibres (see [13], p.94), and the numerical computations were presented in that paper. In particular, if we let $\alpha = 1$ and $\beta = \frac{1}{2}$, then Eq. (1) will be changed to the special form $u_{xt} = u + \frac{1}{6}(u^3)_{xx}$. In [14–20], many results are obtained about the special SPE. In recent works [21–23], we have investigated the dynamical behavior of loop soliton solutions for several equations.

In the present paper, by using Lie group analysis and the generalized symmetry method, we will investigate the short pulse equation in detail, and the exact explicit traveling wave solutions and analytic solutions will be given.

For the sake of Lie symmetry analysis, we write Eq. (1) as the following another usual form in mathematical physics:

$$u_t = \alpha D^{-1} u + \beta u^2 u_x + p, \tag{3}$$

where $D^{-1} = \int \cdot \mathrm{d}x$, $p = p(t)$ is an arbitrary integral function. Moreover, we have

$$u_{tt} = \alpha^2 D^{-1} v + \frac{4}{3}\alpha\beta u^3 + 4\beta^2 u^3 u_x^2 + 2\alpha\beta v u u_x + \beta^2 u^4 u_{xx} + 2\beta p u u_x + r, \tag{4}$$

where $v = D^{-1}u$, $r = r(x, t)$ is an integral function. We note that Eqs. (3) and (4) are necessary for Lie symmetry analysis in what follows.

The outline of this paper is as follows. In Section 2, we perform Lie group analysis for the short pulse equation. In Section 3, the generalized symmetry method was employed for investigating the symmetries of Eq. (1). In Section 4, we will present the qualitative analysis and provide all the traveling wave solutions for this equation. In Section 5, the exact analytic solutions are obtained by using the power series method. In Section 6, we conclude and make some remarks.

2. Lie symmetry analysis for SPE

In this section, we will perform Lie group method for Eq. (1).

The Lie group method is sometimes also called symmetry analysis. Roughly speaking, a symmetry group of a system of differential equations is a group which transforms solutions of the system to other solutions. Once one has determined the symmetry group of a system of differential equations, a number of applications become available. To start with, one can directly use the defining property of such a group and construct new solutions to the system from known ones.

Firstly, let us consider a one-parameter Lie group of infinitesimal transformation:

$$\begin{aligned} x &\to x + \epsilon\xi(x, t, u), \\ t &\to t + \epsilon\tau(x, t, u), \\ u &\to u + \epsilon\phi(x, t, u), \end{aligned}$$

with a small parameter $\epsilon \ll 1$. The vector field associated with the above group of transformations can be written as

$$V = \xi(x, t, u)\frac{\partial}{\partial x} + \tau(x, t, u)\frac{\partial}{\partial t} + \phi(x, t, u)\frac{\partial}{\partial u}. \tag{5}$$

The symmetry group of Eq. (1) will be generated by the vector field of the form (5). Applying the second prolongation $\mathrm{pr}^{(2)}V$ of V to Eq. (2), we find that the coefficient functions ξ, τ and ϕ must satisfy the symmetry condition

$$-\alpha\phi - 2\beta u_x^2\phi - 2\beta u u_{xx}\phi - 4\beta u u_x \phi^x - \beta u^2 \phi^{xx} + \phi^{xt} = 0, \tag{6}$$

where ϕ, ϕ^x, ϕ^{xx} and ϕ^{xt} are all coefficients of $\mathrm{pr}^{(2)}V = \mathrm{pr}^{(1)}V + \phi^{xx}\frac{\partial}{\partial u_{xx}} + \phi^{xt}\frac{\partial}{\partial u_{xt}} + \phi^{tt}\frac{\partial}{\partial u_{tt}}$, and furthermore, we have

$$\phi^x = D_x\phi - u_x D_x\xi - u_t D_x\tau, \tag{7}$$

$$\phi^{xx} = D_x^2\phi - u_x D_x^2\xi - u_t D_x^2\tau - 2u_{xx}D_x\xi - 2u_{xt}D_x\tau, \tag{8}$$

$$\phi^{xt} = D_t D_x\phi - u_x D_t D_x\xi - u_{xt}D_x\xi - u_{xx}D_t\xi - u_t D_t D_x\tau - u_{tt}D_x\tau - u_{xt}D_t\tau, \tag{9}$$

where D_x and D_t are the total derivatives with respect to x and t, respectively.

Substituting (2)–(4) into (7)–(9), respectively, then plugging (7)–(9) into (6), and equating the coefficients of the various monomials in the first, second and the other order partial derivatives with respect to x and various powers of u, we can find the determining equations for the symmetry group of the short pulse equation. Solving these equations, we get the following forms of the coefficient functions

$$\xi = c_1 x + c_3, \qquad \tau = -c_1 t + c_2, \qquad \phi = c_1 u,$$

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