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On periodic solutions for even order differential equations

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Abstract

A class of sufficient conditions are obtained for the existence of a unique 2π-periodic solution of even order differential equations, employing an initial value problem.
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1. Introduction

In this paper, we consider the existence and uniqueness of periodic solutions for the following even order differential system:

$$\left(g(t)u^{(k)}\right)^{(k)} + \sum_{j=1}^{k-1} \alpha_j u^{(2j)} + (-1)^{k+1} h(t,u) = f(t), \tag{1.1}$$

where $t \in [0, 2\pi]$, $u \in \mathfrak{R}^n$, $g \in C^k(\mathfrak{R})$, $f \in C^1(\mathfrak{R}^n)$, $h \in C^1(\mathfrak{R} \times \mathfrak{R}^n)$ are 2π-periodic in t, α_j, $j = 1, \ldots, k-1$, are constants.

Throughout this paper we use the following assumptions:

(A1) The Jacobian matrix $h_u = (h_{iu_j})$ is a symmetric matrix, $g \in C^k(\mathfrak{R})$ satisfies $0 < M_1 \le g(t) \le M_2$ on $\mathfrak{R}$ for some constants M_1 and M_2.

(A2) The eigenvalues $\lambda_i(h_u)$, $i = 1, \ldots, n$, of h_u satisfy

$$\tau(N_i) + \phi_i(t, \|u\|) \le \lambda_i(h_u) \le \omega(N_i + 1) - \varphi_i(t, \|u\|)$$

on $\mathfrak{R} \times \mathfrak{R}^n$, and

$$\tau(N_i) = M_2 N_i^{2k} + \sum_{j=1}^{k-1} (-1)^{j-k} \alpha_j N_i^{2j},$$

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$$\omega(N_i+1) = M_1(N_i+1)^{2k} + \sum_{j=1}^{k-1}(-1)^{j-k}\alpha_j(N_i+1)^{2j}, \quad i = 1,\ldots,n,$$

respectively, where N_i are nonnegative integers, $\phi_i(t,s)$ and $\varphi_i(t,s)$, $i = 1,\ldots,n$, are continuous functions defined from $[0,2\pi]\times[0,\infty)$ to $(0,\infty)$, and nonincreasing with respect to s,

$$\int_0^{+\infty} \min_{1\le i\le n, t\in[0,2\pi]}\{\phi_i(t,s),\varphi_i(t,s)\}\mathrm{d}s = +\infty. \tag{1.2}$$

Without loss of generality, it always is assumed that $\tau(N)$, $\omega(N)$ are positive nondecreasing sequences about N, respectively, and $\tau(N_i) < \tau(N_{i+1})$, $\omega(N_i+1) < \omega(N_{i+1}+1)$.

In the one-dimensional case, that is, when $n = 1$, applying the lemma on bilinear forms [12] and the Leray–Schauder principle [2,7,8,16,18], Cong [3] obtained the following result.

Theorem 1.1. *Let $h \in C^1(\Re\times\Re)$ with $h(t+2\pi,u) = h(t,u)$, and $g \in C^k(\Re)$ satisfy $0 < M_1 \le g(t) \le M_2$ on $\Re$ for some constants M_1 and M_2. Assume that there exist constants a and b, and a nonnegative integer m such that the inequality*

$$0 < M_2 m^{2k} + \sum_{j=1}^{k-1}(-1)^{j-k}\alpha_j m^{2j} < a \le h_u(t,u) \le b < M_1(m+1)^{2k} + \sum_{j=1}^{k-1}(-1)^{j-k}\alpha_j(m+1)^{2j}$$

holds on $\Re\times\Re$ for constants α_j, $j = 1,\ldots,k-1$, in (1.1). Then (1.1) has a unique 2π-periodic solution.

For more results on the existence of periodic solutions of (1.1), we refer the reader to [1,2,4,7,9–12,15–17,19, 21–24].

In this paper, we establish some new sufficient conditions that have connections with an initial value problem for the existence of a unique 2π-periodic solution of (1.1). These conditions are given in Section 3. The results of [1–3,12,19,22–24] are consequences of this paper.

2. Preliminaries and lemmas

Consider the linear operator $L : \mathcal{D}(L) \to \mathcal{X}$ where

$$Lu = (-1)^k\left(g(t)u^{(k)}\right)^{(k)} + (-1)^k\sum_{j=1}^{k-1}\alpha_j u^{(2j)}$$

and a continuously Fréchet differentiable operator $N : \mathcal{D}(L) \to \mathcal{X}$ which is defined by

$$(N(u))(t) = -h(t,u(t)), \quad t\in[0,2\pi]. \tag{2.1}$$

Then (1.1) is reformulated as

$$Lu + N(u) = (-1)^k f. \tag{2.2}$$

Let $\mathcal{X} = \mathcal{L}_n^2[0,2\pi]$ be the set of all vector-valued functions $u(t) = (u_i(t))_{n\times 1}$ on $[0,2\pi]$ such that $u_i \in \mathcal{L}^2[0,2\pi]$ for $i = 1,\ldots,n$. Then $\mathcal{X}$ is a Hilbert space with the following inner product:

$$\langle u,v\rangle = \int_0^{2\pi} u^T(t)v(t)\mathrm{d}t, \tag{2.3}$$

and we denote by $\|\cdot\|$ the norm induced by this inner product. Also

$$\mathcal{D}(L) = \Big\{u(t) = (u_1(t),\ldots,u_n(t))^T \mid u^{(i)}(0) = u^{(i)}(2\pi), i = 0,1,\ldots,2k-1;$$
$$u_i^{(2k-1)}(t) \text{ absolutely continuous on } [0,2\pi] \text{ and } u_i^{(2k)}(t)\in\mathcal{L}^2[0,2\pi]\Big\}, \tag{2.4}$$

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