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Coderivative calculus and metric regularity for constraint and variational systems[☆]

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Abstract

This paper provides new developments in *generalized differentiation* theory of variational analysis with their applications to *metric regularity* of parameterized *constraint* and *variational systems* in finite-dimensional and infinite-dimensional spaces. Our approach to the study of metric regularity for these two major classes of parametric systems is based on appropriate *coderivative* constructions for set-valued mappings and on extended calculus rules supporting their computation and estimation. The main attention is paid in this paper to the so-called *reversed mixed coderivative*, which is of crucial importance for efficient *pointwise characterizations* of metric regularity in the general framework of set-valued mappings between *infinite-dimensional* spaces. We develop new *calculus results* for the latter coderivative that allow us to compute it for large classes of parametric constraint and variational systems. On this basis we derive verifiable *sufficient* conditions, *necessary* conditions as well as *complete characterizations* for metric regularity of such systems with computing the corresponding *exact bounds* of metric regularity constants/moduli. This approach allows us to reveal general settings in which *metric regularity fails* for major classes of parametric variational systems. Furthermore, the developed coderivative calculus leads us also to establishing new formulas for computing the *radius of metric regularity* for constraint and variational systems, which characterize the maximal region of preserving metric regularity under linear (and other types of) *perturbations* and are closely related to *conditioning* aspects of optimization.
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1. Introduction

Metric regularity and closely related notions of *linear openness* and *robust Lipschitzian stability* have been widely recognized to be among the most basic concepts of nonlinear analysis that are crucial from the viewpoints of both theory and applications, especially to problems in optimization, equilibria, control, sensitivity, conditioning,

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etc., see, e.g., [1,4,10,11,18] with the references and commentaries therein. Although these notions have been implicitly originated by the classical results of nonlinear and convex analysis (Lyusternik–Graves theorem, Hoffman inequality, Robinson–Ursescu theorem), their essence and understanding – even for the classical settings – have been fully achieved quite recently in the framework of modern *variational analysis*; see more details in discussions in [4,10,18].

The most significant common feature of the aforementioned notions is a *uniform linear rate* in the underlying relationships, which makes it possible to derive verifiable *qualitative* and *quantitative characterizations* of these notions with precise *computing* the underlying constants. Such characterizations and formulas for the *exact bounds/moduli* have been obtained for general set-valued mappings of closed graph by using appropriate generalized differential constructions of variational analysis, particularly the *coderivative* notion for set-valued mappings introduced by Mordukhovich [7]. We refer the reader to [8,18] for the full account in finite-dimensional spaces and to [1,4,10] for infinite-dimensional extensions, modifications, and commentaries. Other infinite-dimensional characterizations, including those in general metric spaces, can be found in [4] and the references therein.

This paper is devoted to developing the *coderivative approach* to the study of metric regularity and its applications in both finite-dimensional and infinite-dimensional spaces, with the focus on infinite-dimensional settings. It has been well recognized that there are several useful modifications of the basic coderivative construction of [7,10], which all agree in finite dimensions but not in infinite-dimensional (starting with simple Hilbert) spaces.

The primary attention in this paper is paid to the so-called *reversed mixed coderivative* of set-valued mappings, which reduces to the original (normal) coderivative of [7] in finite-dimensional spaces but may be essentially different in infinite dimensions from the latter and its "mixed" version broadly studied and applied in the recent two-volume book [10,11] and numerous publications referred and discussed therein. It happens that the reversed mixed coderivative plays a major role in characterizing metric regularity in infinite-dimensional spaces while being largely underinvestigated from the viewpoint of its calculus and computation/estimation in comparison with the aforementioned normal and mixed counterparts.

It what follows we compute and/or upper estimate the reversed mixed coderivative for large classes of parametric systems that are overwhelmingly involved in variational analysis, optimization, and their applications. These classes of systems are known, respectively, as *parametric constraint systems* (PCS) and *parametric variational systems* (PVS); they particularly include parameterized sets of feasible solutions, optimal solutions, stationary points, etc., in various problems of optimization and equilibria; see more discussions and examples in Sections 3 and 4. Our general framework to compute the reversed mixed coderivative of PCS and PVS in this paper is the class of all *Banach spaces*, but certain more specified results (mainly of the inclusion/upper estimate type under different assumptions) are derived for the class of *Asplund spaces*, which contains, e.g., every reflexive Banach space; see Section 2 for more details.

The coderivative calculus results derived in this paper are then applied to establishing efficient conditions (*necessary, sufficient*, and *complete characterizations*) for *metric regularity* of PCS and PVS with computing/estimating the corresponding regularity *exact bounds/moduli*. In particular, the characterizations obtained in this way allow us to reveal rather general classes of parametric variational systems, which are *not metrically regular*.

Another issue we address in this paper by employing new coderivative calculus results is to compute/estimate the so-called *radius of metric regularity* (describing the area of preserving metric regularity under perturbations) for PCS and PVS in finite and infinite dimensions. This becomes possible due to recently discovered relationships between the exact bound of metric regularity and its radius. The radius of metric regularity is closely connected to the *distance of infeasibility* and other characteristics of *conditioning* in optimization and related areas; see Section 6 and the references therein.

It is worth mentioning that the main *pointbased* results on both coderivative calculus and its applications to metric regularity are derived in what follows by developing appropriate *limiting procedures* from the corresponding "fuzzy" results involving "nonrobust" Fréchet-type constructions. The realization of these procedures requires a careful analysis conducted and specified in this paper for particular classes of parametric systems.

The rest of the paper is organized as follows. In Section 2 we define the basic *generalized differential* constructions of our further study and applications and briefly overview some concepts and facts from variational analysis widely used in the paper. Section 3 is devoted to computing the reversed mixed coderivative of *parametric constraint systems* and their specifications including *implicit multifunctions* and systems of *feasible solutions* to nonlinear programming. In Section 4 we compute and estimate the reversed mixed coderivative of *parametric variational systems* described as

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