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Dynamic bilateral contact problem for viscoelastic piezoelectric materials with adhesion☆

Stanisław Migórski*, Anna Ochal

Jagiellonian University, Faculty of Mathematics and Computer Science, Institute of Computer Science, ul. Nawojki 11, 30072 Krakow, Poland

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Abstract

A model of a dynamic viscoelastic adhesive contact between a piezoelectric body and a deformable foundation is described. The model consists of a system of the hemivariational inequality of hyperbolic type for the displacement, the time dependent elliptic equation for the electric potential and the ordinary differential equation for the adhesion field. In the hemivariational inequality the friction forces are derived from a nonconvex superpotential through the generalized Clarke subdifferential. The existence of a weak solution is proved by embedding the problem into a class of second-order evolution inclusions and by applying a surjectivity result for multivalued operators.

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1. Introduction

The mathematical theory of contact mechanics is concerned with the mathematical structures which underlie general contact problems with different constitutive laws, various geometries and different contact conditions. The aim of this theory is to predict reliably the evolution of contact processes in various situations and to provide a rigorous mathematical background for the constructions of models for contact phenomena. The mathematical analysis of contact problems is based on the fundamental physical principles and requires knowledge from partial differential equations, nonlinear analysis and numerical methods.

In this paper we study a dynamic viscoelastic adhesive contact between a body and a deformable foundation. The contact is bilateral and friction is modeled by a subdifferential boundary condition involving nonsmooth and nonconvex superpotentials. We assume that the mechanical properties of the body are viscoelastic and therefore the behavior of the material is described by a modified Kelvin–Voigt constitutive law which takes into account the piezoelectric effect of the body. The evolution of the adhesive field is described using an additional variable which is governed by an ordinary differential equation on the contact surface.

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* Corresponding author. Tel.: +48 12 632 3355; fax: +48 12 634 1865.
E-mail addresses: migorski@softlab.ii.uj.edu.pl (S. Migórski), ochal@softlab.ii.uj.edu.pl (A. Ochal).

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The novelty of the paper is in dealing with a model which couples the viscoelastic and piezoelectric properties of the material with the adhesive properties on the contact surface and nonmonotone possibly multivalued boundary conditions. Because of the latter the mathematical problem is formulated as a system of the hemivariational inequality of hyperbolic type for the displacement, the time dependent elliptic boundary value problem for the electric potential and the ordinary differential equation for the bonding field.

We mention that the quasistatic contact problems for piezoelectric bodies without adhesion have been considered in [19] and with adhesion in [21,23]. The dynamic problems for viscoelastic piezoelectric materials without adhesion were treated in [7]. On the other hand, dynamic contacts with adhesion and no piezoelectric effects were studied in [3]. Analysis of various models for adhesive contact can be found in [10,11,22,24,25] and the references therein. We underline that the methods used in the aforementioned papers cannot be applied to the model under consideration since the subdifferential boundary conditions are nonmonotone and multivalued. Therefore, we use arguments as in [12,14–18,20] developed in the theory of hemivariational inequalities combined with results for elliptic problems and ordinary differential equations. We provide the existence of a weak solution to the model. We associate with our system a second-order evolution inclusion and apply a surjectivity result for multivalued operators. The question of uniqueness of the solution is left open.

The paper is structured as follows. In Section 2 we introduce the preliminary material. Section 3 is devoted to mechanical and variational formulations of the model. The proof of the main result (cf. Theorem 7) is delivered in Section 4. Finally, in Section 5 we provide examples of a superpotential generating the subdifferential boundary condition and an adhesion field which satisfy our hypotheses.

2. Preliminaries

In this section we introduce the notation and recall definitions needed in the sequel.

Let X be a Banach space with a norm $\|\cdot\|_X$. The dual space of X is denoted by X^* and $\langle\cdot,\cdot\rangle_{X^*\times X}$ is the duality pairing of X and X^*. By $\mathcal{L}(X, X^*)$ we denote the class of linear and bounded operators from X to X^*. For a set $U \subset X$ we define $\|U\|_X = \sup\{\|u\|_X : u \in U\}$.

Let H be a separable Hilbert space and let V be a dense subspace of H carrying the structure of a separable reflexive Banach space with continuous embedding $V \subset H$. Identifying H with its dual, the triple of spaces (V, H, V^*) is called an evolution triple (cf. [6]). Moreover, we assume that the embedding $V \subset H$ is compact (and hence also $H \subset V^*$ compactly).

Given a finite time interval $(0, T)$, $T > 0$ and an evolution triple (V, H, V^*), we define the spaces $\mathcal{V} = L^2(0, T; V)$, $\widehat{\mathcal{H}} = L^2(0, T; H)$ and $\mathcal{W} = \{w \in \mathcal{V} : w' \in \mathcal{V}^*\}$, where the time derivative involved in the definition is understood in the sense of vector valued distributions. We have the following continuous embeddings: $\mathcal{W} \subset \mathcal{V} \subset \widehat{\mathcal{H}} \subset \mathcal{V}^*$. Equipped with the norm $\|v\|_{\mathcal{W}} = \|v\|_{\mathcal{V}} + \|v'\|_{\mathcal{V}^*}$ the space $\mathcal{W}$ becomes a separable reflexive Banach space. It is well known (cf. e.g. [6]) that the space $\mathcal{W}$ is embedded continuously in $C(0, T; H)$ (the space of continuous functions on $[0, T]$ with values in H), i.e. every element of $\mathcal{W}$, after a possible modification on a set of measure zero, has a unique continuous representative in $C(0, T; H)$. Moreover, since V is embedded compactly in H, then so is $\mathcal{W}$ in $\widehat{\mathcal{H}}$ (cf. [6]). The inner products in Hilbert spaces H and $\widehat{\mathcal{H}}$ are denoted by $\langle\cdot,\cdot\rangle_H$ and $\langle\cdot,\cdot\rangle_{\widehat{\mathcal{H}}}$, respectively.

We recall the definitions of the generalized directional derivative and the generalized gradient of Clarke for a locally Lipschitz function $h\colon X \to \mathbb{R}$, where X is a Banach space (see [4]). The generalized directional derivative of h at $x \in X$ in the direction $v \in X$, denoted by $h^0(x; v)$, is defined by

$$h^0(x; v) = \limsup_{y\to x,\ t\downarrow 0} \frac{h(y + tv) - h(y)}{t}.$$

The generalized gradient of h at x, denoted by $\partial h(x)$, is a subset of a dual space X^* given by $\partial h(x) = \{\zeta \in X^* : h^0(x; v) \ge \langle\zeta, v\rangle_{X^*\times X}$ for all $v \in X\}$. A locally Lipschitz function h is called regular (in the sense of Clarke) at $x \in X$ if for all $v \in X$ the one-sided directional derivative $h'(x; v)$ exists and satisfies $h^0(x; v) = h'(x; v)$ for all $v \in X$.

We recall the following notation. We denote by $\mathcal{S}_d$ the linear space of second-order symmetric tensors on $\mathbb{R}^d$, or equivalently, the space $\mathbb{R}_s^{d\times d}$ of symmetric matrices of order d. The inner products and the corresponding norms on $\mathbb{R}^d$ and $\mathcal{S}_d$ are given by

$$u \cdot v = u_i v_i, \qquad \|v\|_{\mathbb{R}^d} = (v \cdot v)^{1/2} \qquad \text{for all } u, v \in \mathbb{R}^d,$$