Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis 68 (2008) 3633–3645
Nonlinear Analysis
www.elsevier.com/locate/na

Global dynamics behaviors for a nonautonomous Lotka–Volterra almost periodic dispersal system with delays☆

Xinzhu Meng[a,b,∗], Jianjun Jiao[c], Lansun Chen[b,d]

[a] *College of Science, Shandong University of Science and Technology, Qingdao 266510, PR China*
[b] *Department of Applied Mathematics, Dalian University of Technology, Dalian 116024, PR China*
[c] *School of Mathematics and Statistics, Guizhou College of Finance and Economics, Guiyang 550004, PR China*
[d] *Institute of Mathematics, Academy of Mathematics and System Sciences, Academia Sinica, Beijing 100080, PR China*

Received 18 June 2006; accepted 5 April 2007

Abstract

A nonautonomous Lotka–Volterra dispersal system with continuous delays and discrete delays is considered. By using a comparison theorem and delay differential equation basic theory, we obtain sufficient conditions for the permanence of the population in every patch. By constructing a suitable Lyapunov functional, we prove that the system is globally asymptotically stable under some appropriate conditions. Using almost periodic functional hull theory, we get sufficient conditions for the existence, uniqueness and globally asymptotical stability for an almost periodic solution. This implies that the population in every patch exhibits stable almost periodic fluctuation. Furthermore, the results show that the permanence and global stability of system, and the existence and uniqueness of a positive almost periodic solution, depend on the delay; then we call it "profitless".

Keywords: Delays; Diffusion; Hull; Global asymptotic stability; Almost periodic solution

1. Introduction

It is well known that the classical Lotka–Volterra type systems form a significant component of the models of species population dynamics. Recently, delays [1–11] and diffusions [11–16] have been extensively introduced into Lotka–Volterra type systems, which enriches the biological background. The effect of environment change in the growth and diffusion of a species in a heterogeneous habitat is a subject of considerable interest in the ecological literature [16–22]. Because of the ecological effects of human activities and industry, more and more habitats are broken into patches and some of them are polluted. In some of these patches, without the contribution from other patches, a species will go to extinction. The general delay differential equations exhibit much more complicated dynamics than ordinary differential equations since a time delay could cause a stable equilibrium to become unstable and cause the population to fluctuate. Negative feedback crowding or the effect of the past life history of the species

☆ This work was supported by the National Natural Science Foundation of China (10471117) and the Natural Science Foundation of SDUST (05g016).
∗ Corresponding author at: College of Science, Shandong University of Science and Technology, Qingdao 266510, PR China.
E-mail address: mxz721106@sdust.edu.cn (X. Meng).

0362-546X/$ - see front matter
doi:10.1016/j.na.2007.04.006

on its present birth rate are common examples illustrating the biological meaning of time delays and justifying their use in these systems.

Since biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically or almost periodically varying environment are considered as important selective forces on systems in a fluctuating environment. So, models in [1–14] take into account both the seasonality of the periodically changing environment and the effects of time delays. The coincidence degree theorem is extensively applied to prove the existence of a periodic solution in [9–11,14]. However, in the real world, it is more realistic to consider an almost periodic system than a periodic system. By using the definition of an almost periodic solution or the contraction mapping and fixed point theory, some authors have done many good works in theory on almost periodic systems [17–19,27–29]. However, there is not much research into global stability of almost periodic systems with time delays based on the construction of suitable Lyapunov functionals and almost periodic hull theory. The most basic and important questions to ask for these systems in the theory of mathematical ecology are those of the permanence, extinctions, global asymptotic behaviors, and existences of coexistence states (for example, the positive equilibrium, strictly positive solution, positive periodic solution, periodic solution and almost periodic solution, etc.) of the population (see [1–22]). In this paper, we consider a nonautonomous Lotka–Volterra almost periodic dispersal system with delays, and investigate the persistence, global asymptotic behaviors, strictly positive solution and strictly positive almost periodic solution of the system by using almost periodic functional hull theory.

Now we shall consider the model with combined effects: diffusion, almost periodicity of the environment and time delays. Namely, we investigate the following nonautonomous Lotka–Volterra type dispersal almost periodic system with discrete and continuous finite time delays which models the diffusion of a single species x_i into n patches connected by discrete dispersal:

$$\begin{aligned}\dot{x}_i(t) = x_i(t)\left[r_i(t) - a_i(t)x_i(t) - a_{ii}(t)x_i(t-\tau_i(t)) - \int_{-\varrho_i}^{0} b_i(t,s)x_i(t+s)\mathrm{d}s\right]\\ + \sum_{j=1}^{n} D_{ij}(t)(x_j(t) - x_i(t)), \quad i = 1,2,\ldots,n,\end{aligned} \tag{1}$$

where $x_i(t)$ represents the density of the species in the ith patch, $D_{ij}(t)$ is the rate of dispersion of the species from patch j to patch i, $r_i(t)$ is the intrinsic growth rate of the species in patch i, $a_i(t)$ is the death rate (or density dependence) of the species in patch i, the terms $a_{ii}(t)x_i(t-\tau_i(t))$ and $\int_{-\varrho_i}^{0} b_i(t,s)x_i(t+s)\mathrm{d}s$ represent the negative feedback crowding and the effect of a period of past life history of the species on its present birth rate, respectively. We refer the readers to good books [23–25] for the basic results on the almost periodic functions.

Suppose that $h(t)$ is an almost periodic function defined on R. Define $h^u = \lim_{t\to\infty}\sup h(t)$, $h^l = \lim_{t\to\infty}\inf h(t)$ and $H(h(t))$ denotes the hull of $h(t)$.

In this paper, for system (1) we always assume that for all $i, j = 1,2,\ldots,n$:

(H_1) The almost periodic functions $r_i(t), a_i(t), a_{ii}(t)$, and $D_{ij}(t)$ are nonnegative and continuous for all $t \in R$, $a_i^l \geq 0, a_{ii}^l \geq 0$ and $a_i^l + a_{ii}^l \neq 0$, $H(r_i(t)) > 0$.

(H_2) The almost periodic functions $b_i(t,s)$ are defined on $R \times [-\varrho_i, 0]$, nonnegative, continuous with respect to $t \in R$, and uniformly integrable with respect to $s \in [-\varrho_i, 0]$ such that $0 < \int_{-\varrho_i}^{0} b_i^l(s)\mathrm{d}s = \alpha_i \leq \beta_i = \int_{-\varrho_i}^{0} b_i^u(s)\mathrm{d}s < \infty$. There are nonnegative and continuous functions $h_i(s)$ defined on $[-\varrho_i, 0]$ satisfying $0 < \int_{-\varrho_i}^{0}(-s)h_i(s)\mathrm{d}s < \infty$ such that $b_i(t,s) \leq h_i(s)$ for all $(t,s) \in R \times [-\varrho_i, 0]$.

(H_3) $\tau_i(t)$ is a nonnegative, continuous and differentiable almost periodic function on R, and $\dot{\tau}_i(t)$ is uniformly continuous with respect to $t \in R$ and $\inf_{t\in R}\{1 - \dot{\tau}_i(t)\} > 0$. Let $\tau^* = \max\{\tau_i^u,\ i = 1,2,\ldots,n\}$; then we have $0 \leq \tau^* < \infty$. Let $\sigma_i(t) = t - \tau_i(t)$; then the function $\sigma_i^{-1}(t)$ is an inverse function for the function $\sigma_i(t)$.

The following notation and concepts are adopted throughout this paper.

Set

$$C_h = \left\{\phi(t) \in C(R_-, R^n) : \int_{-\varrho}^{0} \sup_{s\leq t\leq 0} |\phi(t)|\mathrm{d}s < \infty\right\},$$