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Asymptotic average shadowing property on compact metric spaces

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Abstract

We prove that if for a continuous map f on a compact metric space X, the chain recurrent set, $R(f)$ has more than one chain component, then f does not satisfy the asymptotic average shadowing property. We also show that if a continuous map f on a compact metric space X has the asymptotic average shadowing property and if A is an attractor for f, then A is the single attractor for f and we have $A = R(f)$. We also study diffeomorphisms with asymptotic average shadowing property and prove that if M is a compact manifold which is not finite with $\dim M = 2$, then the C^1 interior of the set of all C^1 diffeomorphisms with the asymptotic average shadowing property is characterized by the set of Ω-stable diffeomorphisms.

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1. Introduction

The shadowing property is an important notion in dynamical systems (see [1]). In [2] Blank introduced the notion of average shadowing property. In [3] Sakai proved that on a closed C^∞ surface, the C^1 interior of the set of all C^1 diffeomorphisms with the average shadowing property is characterized by the set of Anosov diffeomorphisms. In [4] Zhang proved that whenever a homeomorphism f on a compact metric space X has the average shadowing property, every point x in X is chain recurrent:

Theorem (*[4]*). *If a homeomorphism f on a compact metric space X has the average shadowing property, then every point x in X is chain recurrent. Moreover f has only one chain component which is the whole space.*

In [5] Park and Zhang proved that if a continuous surjective map f on a compact metric space X has the average shadowing property, then every point x in X is chain recurrent:

Theorem (*[5]*). *If a continuous surjective map f on a compact metric space X has the average shadowing property, then every point x in X is chain recurrent. Moreover f has only one chain component which is the whole space.*

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Gu [10] introduced a new shadowing property, the asymptotic average shadowing property (AASP), which is weaker than the asymptotic-pseudo-orbit tracing property, and studied the relation between the AASP and transitivity.

Theorem (*[10]*). *Let X be a compact metric space and f be a continuous map from X onto itself. If f has the AASP, then $X = R(f)$, the chain recurrent set of f.*

In this paper first we omit the surjectively property of the continuous map f. In Theorem 1 we prove that if for a continuous map f on a compact metric space X, $R(f)$ has more than one chain component on X, then f does not satisfy the asymptotic average shadowing property. In Theorem 2 we show that if a continuous map f on a compact metric space X has the asymptotic average shadowing property and A is an attractor for f, then A is the single attractor for f and we have $A = R(f)$. In addition we study the diffeomorphisms with asymptotic average shadowing property and prove that if M is a compact manifold with infinite number of elements and $\dim M = 2$, then the C^1 interior of the set of all C^1 diffeomorphisms with the asymptotic average shadowing property is characterized by the set of Ω-stable diffeomorphisms. Finally we state some relations between shadowing property and asymptotic average shadowing property.

2. Notations

Let (X, d) be a compact metric space and let $f : X \to X$ be a homeomorphism of X onto itself. A sequence $\{x_n\}_{n\in\mathbb{Z}}$ is called an orbit of f if for each $n \in \mathbb{Z}$ we have $x_{n+1} = f(x_n)$, it is called a δ-pseudo-orbit of f if for each $n \in \mathbb{Z}$,

$$d(f(x_n), x_{n+1}) \leq \delta.$$

The homeomorphism f is said to have the shadowing property if for every $\epsilon > 0$ there exists $\delta > 0$ such that every δ-pseudo-orbit $\{x_n\}_{n\in\mathbb{Z}}$ is ϵ shadowed by the orbit $\{f^n(y) : n \in \mathbb{Z}\}$, for some y in X, i.e. $d(f^n(y), x_n) < \epsilon$, for all $n \in \mathbb{Z}$.

We denote the set $\{x \in X : x \in \omega_f(x) \cap \alpha_f(x)\}$ by $C(f)$, where $\omega_f(x)$ and $\alpha_f(x)$ are the positive and negative limit sets of x for f, respectively.

Let $x, y \in X$ be given. We write $x \overset{f}{\to} y$ if and only if for every $\delta > 0$ there is a δ-pseudo-orbit $\{x_i\}_{i=0}^{l}$ of some length $l + 1$ of f, such that $x = x_0, \ldots, x_n = y$. We write $x \overset{f}{\sim} y$ if $x \overset{f}{\to} y$ and $y \overset{f}{\to} x$, and let

$$R(f) = \{x \in X : x \overset{f}{\sim} x\}.$$

It is easy to see that $\overset{f}{\sim}$ is an equivalent relation on $R(f)$, the equivalence classes are called chain components of f. For every x in X let

$$A^s(x) := \{y \in X : y \overset{f}{\to} x\}$$
$$A^u(x) := \{y \in X : x \overset{f}{\to} y\}.$$

Let (X, d) be a compact metric space and let f be a continuous map (or homeomorphism) of X into itself. For $\delta > 0$ a sequence $\{x_i\}_{i=0}^{\infty}$ (or $\{x_i\}_{i=-\infty}^{\infty}$) in X is called a δ-average-pseudo-orbit of f if there exists a positive integer $N = N(\delta)$ such that for all $n \geq N$ and $k \in \mathbb{N}$ (or $k \in \mathbb{Z}$)

$$\frac{1}{n}\sum_{i=0}^{n-1} d(f(x_{i+k}), x_{i+k+1}) < \delta.$$

We say that f has the average shadowing property if for every $\epsilon > 0$ there is $\delta > 0$ such that every δ-average-pseudo-orbit $\{x_i\}_{i=0}^{\infty}$ (or $\{x_i\}_{i=-\infty}^{\infty}$) is ϵ-shadowed in average by the orbit of some point $y \in X$, that is

$$\limsup_{n\mapsto\infty} \frac{1}{n}\sum_{i=0}^{n-1} d(f^i(y), x_i) < \epsilon.$$

A sequence $\{x_i\}_{i=0}^{\infty}$ in X is called an asymptotic-average-pseudo-orbit of f if